

Stan Functions Reference

Version 2.38

Stan Development Team



Table of Contents

Overview 1

I Built-in Functions	2
1. Void Functions	4
1.1 Print statement	4
1.2 Reject statement	4
1.3 Fatal error statement	5
2. Integer-Valued Basic Functions	6
2.1 Integer-valued arithmetic operators	6
2.2 Absolute functions	8
2.3 Bound functions	9
2.4 Size functions	9
2.5 Casting functions	10
3. Real-Valued Basic Functions	11
3.1 Vectorization of real-valued functions	11
3.2 Mathematical constants	16
3.3 Special values	17
3.4 Log probability function	17
3.5 Logical functions	18
3.6 Real-valued arithmetic operators	22
3.7 Step-like functions	24
3.8 Power and logarithm functions	26
3.9 Trigonometric functions	28
3.10 Hyperbolic trigonometric functions	29
3.11 Link functions	29
3.12 Probability-related functions	30
3.13 Combinatorial functions	31

3.14	Composed functions	38
3.15	Special functions	41
3.16	Hypergeometric Functions	41
4.	Complex-Valued Basic Functions	43
4.1	Complex assignment and promotion	43
4.2	Complex constructors and accessors	43
4.3	Complex arithmetic operators	44
4.4	Complex comparison operators	46
4.5	Complex (compound) assignment operators	47
4.6	Complex special functions	47
4.7	Complex exponential and power functions	49
4.8	Complex trigonometric functions	50
4.9	Complex hyperbolic trigonometric functions	51
5.	Array Operations	53
5.1	Reductions	53
5.2	Array size and dimension function	57
5.3	Array broadcasting	58
5.4	Array concatenation	60
5.5	Sorting functions	60
5.6	Reversing functions	62
6.	Matrix Operations	63
6.1	Integer-valued matrix size functions	63
6.2	Matrix arithmetic operators	64
6.3	Transposition operator	68
6.4	Elementwise functions	68
6.5	Dot products and specialized products	71
6.6	Reductions	75
6.7	Broadcast functions	78
6.8	Diagonal matrix functions	79
6.9	Container construction functions	80
6.10	Slicing and blocking functions	82
6.11	Matrix and vector concatenation	84

6.12	Special matrix functions	86
6.13	Gaussian Process Covariance Functions	87
6.14	Linear algebra functions and solvers	94
6.15	Sort functions	102
6.16	Reverse functions	104
7.	Complex Matrix Operations	105
7.1	Complex promotion	105
7.2	Integer-valued complex matrix size functions	106
7.3	Complex matrix arithmetic operators	107
7.4	Complex Transposition Operator	111
7.5	Complex elementwise functions	112
7.6	Dot products and specialized products for complex matrices	114
7.7	Complex reductions	116
7.8	Vectorized accessor functions	117
7.9	Complex broadcast functions	118
7.10	Diagonal complex matrix functions	119
7.11	Slicing and blocking functions for complex matrices	119
7.12	Complex matrix concatenation	121
7.13	Complex special matrix functions	123
7.14	Complex linear algebra functions	124
7.15	Reverse functions for complex matrices	128
8.	Sparse Matrix Operations	129
8.1	Compressed row storage	129
8.2	Conversion functions	130
8.3	Sparse matrix arithmetic	131
9.	Mixed Operations	132
10.	Compound Arithmetic and Assignment	140
10.1	Compound addition and assignment	140
10.2	Compound subtraction and assignment	140
10.3	Compound multiplication and assignment	140

10.4	Compound division and assignment	141
10.5	Compound elementwise multiplication and assignment	141
10.6	Compound elementwise division and assignment	141
11.	Higher-Order Functions	142
11.1	Algebraic equation solvers	142
11.2	Ordinary differential equation (ODE) solvers	145
11.3	Differential-Algebraic equation (DAE) solver	150
11.4	1D integrator	152
11.5	Reduce-sum function	155
11.6	Map-rect function	157
12.	Variable Transformation Functions	158
12.1	Transforms for scalars	158
12.2	Transforms for constrained vectors	161
12.3	Transforms for constrained matrices	164
13.	Deprecated Functions	169
13.1	Integer division with operator /	169
13.2	integrate_ode_rk45, integrate_ode_adams, integrate_ode_bdf ODE Integrators	169
13.3	algebra_solver, algebra_solver_newton algebraic solvers	172
14.	Removed Functions	175
14.1	multiply_log and binomial_coefficient_log functions	175
14.2	get_lp() function	175
14.3	fabs function	175
14.4	Exponentiated quadratic covariance functions	175
14.5	Real arguments to logical operators operator&&, operator , and operator!	176
15.	Conventions for Probability Functions	177
15.1	Suffix marks type of function	177
15.2	Argument order and the vertical bar	177
15.3	Sampling notation	177

15.4	Finite inputs	178
15.5	Boundary conditions	178
15.6	Pseudorandom number generators	178
15.7	Cumulative distribution functions	178
15.8	Vectorization	179
II	Discrete Distributions	183
16.	Binary Distributions	185
16.1	Bernoulli distribution	185
16.2	Bernoulli distribution, logit parameterization	186
16.3	Bernoulli-logit generalized linear model (Logistic Regression)	187
17.	Bounded Discrete Distributions	190
17.1	Binomial distribution	190
17.2	Binomial distribution, logit parameterization	191
17.3	Binomial-logit generalized linear model (Logistic Regression)	192
17.4	Beta-binomial distribution	195
17.5	Hypergeometric distribution	196
17.6	Categorical distribution	197
17.7	Categorical logit generalized linear model (softmax regression)	199
17.8	Discrete range distribution	201
17.9	Ordered logistic distribution	202
17.10	Ordered logistic generalized linear model (ordinal regression)	203
17.11	Ordered probit distribution	205
18.	Unbounded Discrete Distributions	207
18.1	Negative binomial distribution	207
18.2	Negative binomial distribution (alternative parameterization)	208

18.3	Negative binomial distribution (log alternative parameterization)	210
18.4	Negative-binomial-2-log generalized linear model (negative binomial regression)	211
18.5	Poisson distribution	213
18.6	Poisson distribution, log parameterization	214
18.7	Poisson-log generalized linear model (Poisson regression)	215
18.8	Beta negative binomial distribution	217
19.	Multivariate Discrete Distributions	219
19.1	Multinomial distribution	219
19.2	Multinomial distribution, logit parameterization	220
19.3	Dirichlet-multinomial distribution	221
III	Continuous Distributions	223
20.	Unbounded Continuous Distributions	225
20.1	Normal distribution	225
20.2	Normal-id generalized linear model (linear regression)	228
20.3	Exponentially modified normal distribution	231
20.4	Skew normal distribution	233
20.5	Student-t distribution	234
20.6	Cauchy distribution	235
20.7	Double exponential (Laplace) distribution	237
20.8	Logistic distribution	238
20.9	Gumbel distribution	240
20.10	Skew double exponential distribution	241
21.	Positive Continuous Distributions	243
21.1	Lognormal distribution	243
21.2	Chi-square distribution	244
21.3	Inverse chi-square distribution	245
21.4	Scaled inverse chi-square distribution	246
21.5	Exponential distribution	247
21.6	Gamma distribution	248

21.7	Inverse gamma Distribution	250
21.8	Weibull distribution	251
21.9	Frechet distribution	252
21.10	Rayleigh distribution	253
21.11	Log-logistic distribution	254
22.	Positive Lower-Bounded Distributions	256
22.1	Pareto distribution	256
22.2	Pareto type 2 distribution	257
22.3	Wiener First Passage Time Distribution	258
23.	Continuous Distributions on $[0, 1]$	265
23.1	Beta distribution	265
23.2	Beta proportion distribution	266
24.	Circular Distributions	268
24.1	Von Mises distribution	268
25.	Bounded Continuous Distributions	271
25.1	Uniform distribution	271
26.	Distributions over Unbounded Vectors	273
26.1	Multivariate normal distribution	273
26.2	Multivariate normal distribution, precision parameterization	275
26.3	Multivariate normal distribution, Cholesky parameterization	277
26.4	Multivariate Gaussian process distribution	279
26.5	Multivariate Gaussian process distribution, Cholesky parameterization	280
26.6	Multivariate Student-t distribution	281
26.7	Multivariate Student-t distribution, Cholesky parameterization	283
26.8	Gaussian dynamic linear models	285
27.	Simplex Distributions	287
27.1	Dirichlet distribution	287

28. Correlation Matrix Distributions	290
28.1 LKJ correlation distribution	290
28.2 Cholesky LKJ correlation distribution	291
29. Covariance Matrix Distributions	293
29.1 Wishart distribution	293
29.2 Wishart distribution, Cholesky Parameterization	294
29.3 Inverse Wishart distribution	295
29.4 Inverse Wishart distribution, Cholesky Parameterization	296
IV Additional Distributions	298
30. Hidden Markov Models	300
30.1 Stan functions	300
V Appendix	302
31. Mathematical Functions	304
31.1 Beta	304
31.2 Incomplete beta	304
31.3 Gamma	304
31.4 Digamma	305
References	306

Overview

This is the reference for the functions defined in the [Stan math library](#) and available in the [Stan](#) programming language.

For more information the Stan language and inference engines and how to use Stan for Bayesian inference, see

- the [Stan User's Guide](#). The Stan user's guide provides example models and programming techniques for coding statistical models in Stan. It also serves as an example-driven introduction to Bayesian modeling and inference;
- the [Stan Reference Manual](#). Stan's modeling language is shared across all of its interfaces. The Stan Language Reference Manual provides a concise definition of the language syntax for all elements in the language together with an overview of the inference algorithms and posterior inference tools.

Copyright and trademark

- Copyright 2011–2025, Stan Development Team and their assignees.
- The Stan name and logo are registered trademarks of NumFOCUS.

Licensing

- *Text content*: [CC-BY ND 4.0 license](#)
- *Computer code*: [BSD 3-clause license](#)
- *Logo*: [Stan logo usage guidelines](#)

Part I

Built-in Functions

1. Void Functions

Stan supports a few special statements for printing or for signaling an issue with the program.

Although `print`, `reject`, and `fatal_error` appear to have the syntax of functions, they are actually special kinds of statements with slightly different form and behavior than other functions. First, they are the constructs that allow a variable number of arguments. Second, they are the only constructs to accept string literals (e.g., "hello world") as arguments. Third, they have no effect on the log density function and operate solely through side effects.

The special keyword `void` is used for their return type because they behave like variadic functions with `void` return type, even though they are special kinds of statements built in to the language.

1.1. Print statement

Printing has no effect on the model's log probability function. Its sole purpose is the side effect (i.e., an effect not represented in a return value) of arguments being printed to whatever the standard output stream is connected to (e.g., the terminal in command-line Stan or the R console in RStan).

```
void print(T1 x1, ..., TN xN)
```

Print the values denoted by the arguments `x1` through `xN` on the output message stream. There are no spaces between items in the `print`, but a line feed (LF; Unicode U+000A; C++ literal '\n') is inserted at the end of the printed line. The types `T1` through `TN` can be any of Stan's built-in numerical types or double quoted strings of characters (bytes).

Available since 2.1

1.2. Reject statement

The `reject` statement has the same syntax as the `print` statement, accepting an arbitrary number of arguments of any type (including string literals). The effect of executing a `reject` statement is to throw an exception internally that terminates the current iteration with a rejection (the behavior of which will depend on the algorithmic context in which it occurs).

```
void reject(T1 x1, ..., TN xN)
```

Reject the current iteration and print the values denoted by the arguments x_1 through x_N on the output message stream. There are no spaces between items in the print, but a line feed (LF; Unicode U+000A; C++ literal '\n') is inserted at the end of the printed line. The types T_1 through T_N can be any of Stan's built-in numerical types or double quoted strings of characters (bytes).

Available since 2.18

1.3. Fatal error statement

The fatal error statement has the same syntax as the print and reject statements, accepting an arbitrary number of arguments of any type (including string literals). The effect of executing a fatal_error statement is to throw an exception internally that terminates the algorithm completely. It can be viewed as an unrecoverable version of reject, and as such should be used only when exiting the algorithm is the only option.

```
void fatal_error(T1 x1, ..., TN xN)
```

Print the values denoted by the arguments x_1 through x_N on the output message stream and then exit the currently running algorithm. There are no spaces between items in the print, but a line feed (LF; Unicode U+000A; C++ literal '\n') is inserted at the end of the printed line. The types T_1 through T_N can be any of Stan's built-in numerical types or double quoted strings of characters (bytes).

Available since 2.35

2. Integer-Valued Basic Functions

This chapter describes Stan's built-in function that take various types of arguments and return integer values.

2.1. Integer-valued arithmetic operators

Stan's arithmetic is based on standard double-precision C++ integer and floating-point arithmetic. If the arguments to an arithmetic operator are both integers, as in $2 + 2$, integer arithmetic is used. If one argument is an integer and the other a floating-point value, as in $2.0 + 2$ and $2 + 2.0$, then the integer is promoted to a floating point value and floating-point arithmetic is used.

Integer arithmetic behaves slightly differently than floating point arithmetic. The first difference is how overflow is treated. If the sum or product of two integers overflows the maximum integer representable, the result is an undesirable wraparound behavior at the bit level. If the integers were first promoted to real numbers, they would not overflow a floating-point representation. There are no extra checks in Stan to flag overflows, so it is up to the user to make sure it does not occur.

Secondly, because the set of integers is not closed under division and there is no special infinite value for integers, integer division implicitly rounds the result. If both arguments are positive, the result is rounded down. For example, $1 / 2$ evaluates to 0 and $5 / 3$ evaluates to 1.

If one of the integer arguments to division is negative, the latest C++ specification (C++11), requires rounding toward zero. This would have $1 / 2$ and $-1 / 2$ evaluate to 0, $-7 / 2$ evaluate to -3, and $7 / 2$ evaluate to 3. Before the C++11 specification, the behavior was platform dependent, allowing rounding up or down. All compilers recent enough to be able to deal with Stan's templating should follow the C++11 specification, but it may be worth testing if you are not sure and plan to use integer division with negative values.

Unlike floating point division, where $1.0 / 0.0$ produces the special positive infinite value, integer division by zero, as in $1 / 0$, has undefined behavior in the C++ standard. For example, the clang++ compiler on Mac OS X returns 3764, whereas the g++ compiler throws an exception and aborts the program with a warning. As with overflow, it is up to the user to make sure integer divide-by-zero does not occur.

Binary infix operators

Operators are described using the C++ syntax. For instance, the binary operator of addition, written `X + Y`, would have the Stan signature `int operator+(int, int)` indicating it takes two real arguments and returns a real value. As noted previously, the value of integer division is platform-dependent when rounding is platform dependent before C++11; the descriptions below provide the C++11 definition.

`int operator+(int x, int y)`

The sum of the addends x and y

$$\text{operator+}(x, y) = (x + y)$$

Available since 2.0

`int operator-(int x, int y)`

The difference between the minuend x and subtrahend y

$$\text{operator-}(x, y) = (x - y)$$

Available since 2.0

`int operator*(int x, int y)`

The product of the factors x and y

$$\text{operator*}(x, y) = (x \times y)$$

Available since 2.0

`int operator/(int x, int y)`

The integer quotient of the dividend x and divisor y

$$\text{operator}/(x, y) = \begin{cases} \lfloor x/y \rfloor & \text{if } x/y \geq 0 \\ -\lfloor \text{floor}(-x/y) \rfloor & \text{if } x/y < 0. \end{cases}$$

deprecated; - use `operator%/%` instead.

Available since 2.0, deprecated in 2.24

`int operator%/(int x, int y)`

The integer quotient of the dividend x and divisor y

$$\text{operator%}/(x, y) = \begin{cases} \lfloor x/y \rfloor & \text{if } x/y \geq 0 \\ -\lfloor \text{floor}(-x/y) \rfloor & \text{if } x/y < 0. \end{cases}$$

Available since 2.24

int operator%(int x, int y)

x modulo y, which is the positive remainder after dividing x by y. If both x and y are non-negative, so is the result; otherwise, the sign of the result is platform dependent.

$$\text{operator\%}(x, y) = x \bmod y = x - y * \lfloor x/y \rfloor$$

Available since 2.13

Unary prefix operators

int operator-(int x)

The negation of the subtrahend x

$$\text{operator-}(x) = -x$$

Available since 2.0

T operator-(T x)

Vectorized version of operator-. If T x is a (possibly nested) array of integers, -x is the same shape array where each individual integer is negated.

Available since 2.31

int operator+(int x)

This is a no-op.

$$\text{operator+}(x) = x$$

Available since 2.0

2.2. Absolute functions

T abs(T x)

The absolute value of x.

This function works elementwise over containers such as vectors. Given a type T which is `int`, or an array of `ints`, `abs` returns the same type where each element has had its absolute value taken.

Available since 2.0, vectorized in 2.30

int int_step(int x)

`int int_step(real x)`

Return the step function of x as an integer,

$$\text{int_step}(x) = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x \leq 0 \text{ or } x \text{ is } \text{NaN} \end{cases}$$

Warning: `int_step(0)` and `int_step(NaN)` return 0 whereas `step(0)` and `step(NaN)` return 1.

See the warning in section [step functions](#) about the dangers of step functions applied to anything other than data.

Available since 2.0

2.3. Bound functions

`int min(int x, int y)`

Return the minimum of x and y .

$$\text{min}(x, y) = \begin{cases} x & \text{if } x < y \\ y & \text{otherwise} \end{cases}$$

Available since 2.0

`int max(int x, int y)`

Return the maximum of x and y .

$$\text{max}(x, y) = \begin{cases} x & \text{if } x > y \\ y & \text{otherwise} \end{cases}$$

Available since 2.0

2.4. Size functions

`int size(int x)`

`int size(real x)`

Return the size of x which for scalar-valued x is 1

Available since 2.26

2.5. Casting functions

It is possible to cast real numbers to integers as long as the real value is data. See [data only qualifiers](#) in the Stan Reference Manual.

`int to_int(data real x)`

Return the value x truncated to an integer. This will throw an error if the value of x is too big to represent as a 32-bit signed integer.

This is similar to `trunc` (see [Rounding functions](#)) but the return type is of type `int`. For example, `to_int(3.9)` is 3, and `to_int(-3.9)` is -3.

Available since 2.31

`I to_int(data T x)`

The vectorized version of `to_int`. This function accepts a (possibly nested) array of reals and returns an array of the same shape where each element has been truncated to an integer.

Available since 2.31

3. Real-Valued Basic Functions

This chapter describes built-in functions that take zero or more real or integer arguments and return real values.

3.1. Vectorization of real-valued functions

Although listed in this chapter, many of Stan's built-in functions are vectorized so that they may be applied to any argument type. The vectorized form of these functions is not any faster than writing an explicit loop that iterates over the elements applying the function—it's just easier to read and write and less error prone.

Unary function vectorization

Many of Stan's unary functions can be applied to any argument type. For example, the exponential function, `exp`, can be applied to `real` arguments or arrays of `real` arguments. Other than for integer arguments, the result type is the same as the argument type, including dimensionality and size. Integer arguments are first promoted to `real` values, but the result will still have the same dimensionality and size as the argument.

Real and real array arguments

When applied to a simple `real` value, the result is a `real` value. When applied to arrays, vectorized functions like `exp()` are defined elementwise. For example,

```
// declare some variables for arguments
real x0;
array[5] real x1;
array[4, 7] real x2;
// ...
// declare some variables for results
real y0;
array[5] real y1;
array[4, 7] real y2;
// ...
// calculate and assign results
y0 = exp(x0);
y1 = exp(x1);
y2 = exp(x2);
```

When `exp` is applied to an array, it applies elementwise. For example, the statement above,

```
y2 = exp(x2);
```

produces the same result for `y2` as the explicit loop

```
for (i in 1:4) {
  for (j in 1:7) {
    y2[i, j] = exp(x2[i, j]);
  }
}
```

Vector and matrix arguments

Vectorized functions also apply elementwise to vectors and matrices. For example,

```
vector[5] xv;
row_vector[7] xrv;
matrix[10, 20] xm;

vector[5] yv;
row_vector[7] yrv;
matrix[10, 20] ym;

yv = exp(xv);
yrv = exp(xrv);
ym = exp(xm);
```

Arrays of vectors and matrices work the same way. For example,

```
array[12] matrix[17, 93] u;

array[12] matrix[17, 93] z;

z = exp(u);
```

After this has been executed, `z[i, j, k]` will be equal to `exp(u[i, j, k])`.

Integer and integer array arguments

Integer arguments are promoted to real values in vectorized unary functions. Thus if `n` is of type `int`, `exp(n)` is of type `real`. Arrays work the same way, so that if `n2` is a one dimensional array of integers, then `exp(n2)` will be a one-dimensional array of reals with the same number of elements as `n2`. For example,

```
array[23] int n1;
array[23] real z1;
z1 = exp(n1);
```

It would be illegal to try to assign `exp(n1)` to an array of integers; the return type is a real array.

Binary function vectorization

Like the unary functions, many of Stan's binary functions have been vectorized, and can be applied elementwise to combinations of both scalars or container types.

Scalar and scalar array arguments

When applied to two scalar values, the result is a scalar value. When applied to two arrays, or combination of a scalar value and an array, vectorized functions like `pow()` are defined elementwise. For example,

```
// declare some variables for arguments
real x00;
real x01;
array[5] real x10;
array[5] real x11;
array[4, 7] real x20;
array[4, 7] real x21;
// ...
// declare some variables for results
real y0;
array[5] real y1;
array[4, 7] real y2;
// ...
// calculate and assign results
y0 = pow(x00, x01);
y1 = pow(x10, x11);
y2 = pow(x20, x21);
```

When `pow` is applied to two arrays, it applies elementwise. For example, the statement above,

```
y2 = pow(x20, x21);
```

produces the same result for `y2` as the explicit loop

```
for (i in 1:4) {
  for (j in 1:7) {
    y2[i, j] = pow(x20[i, j], x21[i, j]);
  }
}
```

Alternatively, if a combination of an array and a scalar are provided, the scalar value is broadcast to be applied to each value of the array. For example, the following statement:

```
y2 = pow(x20, x00);
```

produces the same result for `y2` as the explicit loop:

```
for (i in 1:4) {
  for (j in 1:7) {
    y2[i, j] = pow(x20[i, j], x00);
  }
}
```

Vector and matrix arguments

Vectorized binary functions also apply elementwise to vectors and matrices, and to combinations of these with scalar values. For example,

```
real x00;
vector[5] xv00;
vector[5] xv01;
row_vector[7] xrv;
matrix[10, 20] xm;

vector[5] yv;
row_vector[7] yrv;
matrix[10, 20] ym;

yv = pow(xv00, xv01);
yrv = pow(xrv, x00);
ym = pow(x00, xm);
```

Arrays of vectors and matrices work the same way. For example,

```
array[12] matrix[17, 93] u;
```

```
array[12] matrix[17, 93] z;
z = pow(u, x00);
```

After this has been executed, $z[i, j, k]$ will be equal to $\text{pow}(u[i, j, k], x00)$.

Input & return types

Vectorised binary functions require that both inputs, unless one is a real, be containers of the same type and size. For example, the following statements are legal:

```
vector[5] xv;
row_vector[7] xrv;
matrix[10, 20] xm;

vector[5] yv = pow(xv, xv)
row_vector[7] yrv = pow(xrv, xrv)
matrix[10, 20] = pow(xm, xm)
```

But the following statements are not:

```
vector[5] xv;
vector[7] xv2;
row_vector[5] xrv;

// Cannot mix different types
vector[5] yv = pow(xv, xrv)

// Cannot mix different sizes of the same type
vector[5] yv = pow(xv, xv2)
```

While the vectorized binary functions generally require the same input types, the only exception to this is for binary functions that require one input to be an integer and the other to be a real (e.g., `bessel_first_kind`). For these functions, one argument can be a container of any type while the other can be an integer array, as long as the dimensions of both are the same. For example, the following statements are legal:

```
vector[5] xv;
matrix[5, 5] xm;
array[5] int xi;
array[5, 5] int xii;
```

```
vector[5] yv = bessel_first_kind(xi, xv);
matrix[5, 5] ym = bessel_first_kind(xii, xm);
```

Whereas these are not:

```
vector[5] xv;
matrix[5, 5] xm;
array[7] int xi;

// Dimensions of containers do not match
vector[5] yv = bessel_first_kind(xi, xv);

// Function requires first argument be an integer type
matrix[5, 5] ym = bessel_first_kind(xm, xm);
```

3.2. Mathematical constants

Constants are represented as functions with no arguments and must be called as such. For instance, the mathematical constant π must be written in a Stan program as `pi()`.

`real pi()`

π , the ratio of a circle's circumference to its diameter

Available since 2.0

`real e()`

e , the base of the natural logarithm

Available since 2.0

`real sqrt2()`

The square root of 2

Available since 2.0

`real log2()`

The natural logarithm of 2

Available since 2.0

`real log10()`

The natural logarithm of 10

Available since 2.0

3.3. Special values

`real not_a_number()`

Not-a-number, a special non-finite real value returned to signal an error

Available since 2.0

`real positive_infinity()`

Positive infinity, a special non-finite real value larger than all finite numbers

Available since 2.0

`real negative_infinity()`

Negative infinity, a special non-finite real value smaller than all finite numbers

Available since 2.0

`real machine_precision()`

The smallest number x such that $(x + 1) \neq 1$ in floating-point arithmetic on the current hardware platform

Available since 2.0

3.4. Log probability function

The basic purpose of a Stan program is to compute a log probability function and its derivatives. The log probability function in a Stan model outputs the log density on the unconstrained scale. A log probability accumulator starts at zero and is then incremented in various ways by a Stan program. The variables are first transformed from unconstrained to constrained, and the log Jacobian determinant added to the log probability accumulator. Then the model block is executed on the constrained parameters, with each sampling statement (~) and log probability increment statement (increment_log_prob) adding to the accumulator. At the end of the model block execution, the value of the log probability accumulator is the log probability value returned by the Stan program.

Stan provides a special built-in function `target()` that takes no arguments and returns the current value of the log probability accumulator. This function is primarily useful for debugging purposes, where for instance, it may be used with a print statement to display the log probability accumulator at various stages of execution to see where it becomes ill defined.

`real target()`

Return the current value of the log probability accumulator.

Available since 2.10

target acts like a function ending in `_lp`, meaning that it may only be used in the model block.

3.5. Logical functions

Like C++, BUGS, and R, Stan uses 0 to encode false, and 1 to encode true. Stan supports the usual boolean comparison operations and boolean operators. These all have the same syntax and precedence as in C++; for the full list of operators and precedences, see the reference manual.

Comparison operators

All comparison operators return boolean values, either 0 or 1. Each operator has two signatures, one for integer comparisons and one for floating-point comparisons. Comparing an integer and real value is carried out by first promoting the integer value.

`int operator<(int x, int y)`

`int operator<(real x, real y)`

Return 1 if x is less than y and 0 otherwise.

$$\text{operator}<(x, y) = \begin{cases} 1 & \text{if } x < y \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

`int operator<=(int x, int y)`

`int operator<=(real x, real y)`

Return 1 if x is less than or equal y and 0 otherwise.

$$\text{operator}<=(x, y) = \begin{cases} 1 & \text{if } x \leq y \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

`int operator>(int x, int y)`

`int operator>(real x, real y)`

Return 1 if x is greater than y and 0 otherwise.

$$\text{operator}>(x, y) = \begin{cases} 1 & \text{if } x > y \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

`int operator>=(int x, int y)`

`int operator>=(real x, real y)`

Return 1 if x is greater than or equal to y and 0 otherwise.

$$\text{operator}>=(x, y) = \begin{cases} 1 & \text{if } x \geq y \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

`int operator==(int x, int y)`

`int operator==(real x, real y)`

Return 1 if x is equal to y and 0 otherwise.

$$\text{operator}==(x, y) = \begin{cases} 1 & \text{if } x = y \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

`int operator!=(int x, int y)`

`int operator!=(real x, real y)`

Return 1 if x is not equal to y and 0 otherwise.

$$\text{operator}!=(x, y) = \begin{cases} 1 & \text{if } x \neq y \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

Boolean operators

Boolean operators return either 0 for false or 1 for true. Inputs may be any real or integer values, with non-zero values being treated as true and zero values treated as false. These operators have the usual precedences, with negation (not) binding the most tightly, conjunction the next and disjunction the weakest; all of the operators bind more tightly than the comparisons. Thus an expression such as `!a && b` is interpreted as `(!a) && b`, and `a < b || c >= d && e != f` as `(a < b) || (((c >= d) && (e != f)))`.

`int operator!(int x)`

Return 1 if x is zero and 0 otherwise.

$$\operatorname{operator}!(x) = \begin{cases} 0 & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$$

Available since 2.0

`int operator!(real x)`

Return 1 if x is zero and 0 otherwise.

$$\operatorname{operator}!(x) = \begin{cases} 0 & \text{if } x \neq 0.0 \\ 1 & \text{if } x = 0.0 \end{cases}$$

deprecated; - use `operator==` instead.

Available since 2.0, deprecated in 2.31

`int operator&&(int x, int y)`

Return 1 if x is unequal to 0 and y is unequal to 0.

$$\operatorname{operator\&\&}(x, y) = \begin{cases} 1 & \text{if } x \neq 0 \text{ and } y \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

`int operator&&(real x, real y)`

Return 1 if x is unequal to 0.0 and y is unequal to 0.0.

$$\operatorname{operator\&\&}(x, y) = \begin{cases} 1 & \text{if } x \neq 0.0 \text{ and } y \neq 0.0 \\ 0 & \text{otherwise} \end{cases}$$

deprecated

Available since 2.0, deprecated in 2.31

int operator|| (int x, int y)

Return 1 if x is unequal to 0 or y is unequal to 0.

$$\text{operator}\ |\ |(x, y) = \begin{cases} 1 & \text{if } x \neq 0 \text{ or } y \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

int operator|| (real x, real y)

Return 1 if x is unequal to 0.0 or y is unequal to 0.0.

$$\text{operator}\ |\ |(x, y) = \begin{cases} 1 & \text{if } x \neq 0.0 \text{ or } y \neq 0.0 \\ 0 & \text{otherwise} \end{cases}$$

deprecated

Available since 2.0, deprecated in 2.31

Boolean operator short circuiting

Like in C++, the boolean operators `&&` and `||` and are implemented to short circuit directly to a return value after evaluating the first argument if it is sufficient to resolve the result. In evaluating `a || b`, if `a` evaluates to a value other than zero, the expression returns the value 1 without evaluating the expression `b`. Similarly, evaluating `a && b` first evaluates `a`, and if the result is zero, returns 0 without evaluating `b`.

Logical functions

The logical functions introduce conditional behavior functionally and are primarily provided for compatibility with BUGS and JAGS.

real step(real x)

Return 1 if x is positive and 0 otherwise.

$$\text{step}(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1 & \text{otherwise} \end{cases}$$

Warning: `int_step(0)` and `int_step(NaN)` return 0 whereas `step(0)` and `step(NaN)` return 1.

The step function is often used in BUGS to perform conditional operations. For instance, `step(a-b)` evaluates to 1 if `a` is greater than `b` and evaluates to 0 otherwise. `step` is a step-like functions; see the warning in section [step functions](#) applied to expressions dependent on parameters.

Available since 2.0

`int is_inf(real x)`

Return 1 if `x` is infinite (positive or negative) and 0 otherwise.

Available since 2.5

`int is_nan(real x)`

Return 1 if `x` is NaN and 0 otherwise.

Available since 2.5

Care must be taken because both of these indicator functions are step-like and thus can cause discontinuities in gradients when applied to parameters; see section [step-like functions](#) for details.

3.6. Real-valued arithmetic operators

The arithmetic operators are presented using C++ notation. For instance `operator+(x,y)` refers to the binary addition operator and `operator-(x)` to the unary negation operator. In Stan programs, these are written using the usual infix and prefix notations as `x + y` and `-x`, respectively.

Binary infix operators

`real operator+(real x, real y)`

Return the sum of `x` and `y`.

$$(x + y) = \text{operator+}(x, y) = x + y$$

Available since 2.0

`real operator-(real x, real y)`

Return the difference between `x` and `y`.

$$(x - y) = \text{operator-}(x, y) = x - y$$

Available since 2.0

real operator*(real x, real y)

Return the product of x and y.

$$(x * y) = \text{operator}^*(x, y) = xy$$

Available since 2.0

real operator/(real x, real y)

Return the quotient of x and y.

$$(x / y) = \text{operator} / (x, y) = \frac{x}{y}$$

Available since 2.0

real operator^(real x, real y)

Return x raised to the power of y.

$$(x^y) = \text{operator}^{\wedge}(x, y) = x^y$$

Available since 2.5

Unary prefix operators

real operator-(real x)

Return the negation of the subtrahend x.

$$\text{operator}-(x) = (-x)$$

Available since 2.0

T operator-(T x)

Vectorized version of operator-. If T x is a (possibly nested) array of reals, -x is the same shape array where each individual number is negated.

Available since 2.31

real operator+(real x)

Return the value of x.

$$\text{operator}+(x) = x$$

Available since 2.0

3.7. Step-like functions

Warning: These functions can seriously hinder sampling and optimization efficiency for gradient-based methods (e.g., NUTS, HMC, BFGS) if applied to parameters (including transformed parameters and local variables in the transformed parameters or model block). The problem is that they break gradients due to discontinuities coupled with zero gradients elsewhere. They do not hinder sampling when used in the data, transformed data, or generated quantities blocks.

Absolute value functions

T abs(T x)

The absolute value of x.

This function works elementwise over containers such as vectors. Given a type T which is `real` `vector`, `row_vector`, `matrix`, or an array of those types, `abs` returns the same type where each element has had its absolute value taken.

Available since 2.0, vectorized in 2.30

real fdim(real x, real y)

Return the positive difference between x and y, which is $x - y$ if x is greater than y and 0 otherwise; see warning above.

$$\text{fdim}(x, y) = \begin{cases} x - y & \text{if } x \geq y \\ 0 & \text{otherwise} \end{cases}$$

Available since 2.0

R fdim(T1 x, T2 y)

Vectorized implementation of the `fdim` function

Available since 2.25

Bounds functions

real fmin(real x, real y)

Return the minimum of x and y; see warning above.

$$\text{fmin}(x, y) = \begin{cases} x & \text{if } x \leq y \\ y & \text{otherwise} \end{cases}$$

Available since 2.0

R fmin(T1 x, T2 y)

Vectorized implementation of the `fmin` function

Available since 2.25

real `fmax`(real x, real y)

Return the maximum of x and y; see warning above.

$$\text{fmax}(x, y) = \begin{cases} x & \text{if } x \geq y \\ y & \text{otherwise} \end{cases}$$

Available since 2.0

R `fmax`(T1 x, T2 y)

Vectorized implementation of the `fmax` function

Available since 2.25

Arithmetic functions

real `fmod`(real x, real y)

Return the real value remainder after dividing x by y; see warning above.

$$\text{fmod}(x, y) = x - \left\lfloor \frac{x}{y} \right\rfloor y$$

The operator $\lfloor u \rfloor$ is the floor operation; see below.

Available since 2.0

R `fmod`(T1 x, T2 y)

Vectorized implementation of the `fmod` function

Available since 2.25

Rounding functions

Warning: Rounding functions convert real values to integers. Because the output is an integer, any gradient information resulting from functions applied to the integer is not passed to the real value it was derived from. With MCMC sampling using HMC or NUTS, the MCMC acceptance procedure will correct for any error due to poor gradient calculations, but the result is likely to be reduced acceptance probabilities and less efficient sampling.

The rounding functions cannot be used as indices to arrays because they return real values. For operations over data or in the generated `quantities` block, the [to_int\(\) function](#) can be used.

R `floor`(T x)

The floor of x, which is the largest integer less than or equal to x, converted to a real value; see warning at start of section [step-like functions](#)

Available since 2.0, vectorized in 2.13

R ceil(T x)

The ceiling of x , which is the smallest integer greater than or equal to x , converted to a real value; see warning at start of section [step-like functions](#)

Available since 2.0, vectorized in 2.13

R round(T x)

The nearest integer to x , converted to a real value; see warning at start of section [step-like functions](#)

Available since 2.0, vectorized in 2.13

R trunc(T x)

The integer nearest to but no larger in magnitude than x , converted to a double value; see warning at start of section [step-like functions](#)

Available since 2.0, vectorized in 2.13

3.8. Power and logarithm functions

R sqrt(T x)

The square root of x

Available since 2.0, vectorized in 2.13

R cbrt(T x)

The cube root of x

Available since 2.0, vectorized in 2.13

R square(T x)

The square of x

Available since 2.0, vectorized in 2.13

R exp(T x)

The natural exponential of x

Available since 2.0, vectorized in 2.13

R exp2(T x)

The base-2 exponential of x

Available since 2.0, vectorized in 2.13

R `log`(T x)

The natural logarithm of x

Available since 2.0, vectorized in 2.13

R `log2`(T x)

The base-2 logarithm of x

Available since 2.0, vectorized in 2.13

R `log10`(T x)

The base-10 logarithm of x

Available since 2.0, vectorized in 2.13

real `pow`(real x, real y)

Return x raised to the power of y.

$$\text{pow}(x, y) = x^y$$

Available since 2.0

R `pow`(T1 x, T2 y)

Vectorized implementation of the pow function

Available since 2.25

R `inv`(T x)

The inverse of x

Available since 2.0, vectorized in 2.13

R `inv_sqrt`(T x)

The inverse of the square root of x

Available since 2.0, vectorized in 2.13

R `inv_square`(T x)

The inverse of the square of x

Available since 2.0, vectorized in 2.13

3.9. Trigonometric functions

real hypot(real x, real y)

Return the length of the hypotenuse of a right triangle with sides of length x and y.

$$\text{hypot}(x, y) = \begin{cases} \sqrt{x^2 + y^2} & \text{if } x, y \geq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Available since 2.0

R hypot(T x, T2 y)

Vectorized implementation of the hypot function

Available since 2.25

R cos(T x)

The cosine of the angle x (in radians)

Available since 2.0, vectorized in 2.13

R sin(T x)

The sine of the angle x (in radians)

Available since 2.0, vectorized in 2.13

R tan(T x)

The tangent of the angle x (in radians)

Available since 2.0, vectorized in 2.13

R acos(T x)

The principal arc (inverse) cosine (in radians) of x

Available since 2.0, vectorized in 2.13

R asin(T x)

The principal arc (inverse) sine (in radians) of x

Available since 2.0

R atan(T x)

The principal arc (inverse) tangent (in radians) of x, with values from $-\pi/2$ to $\pi/2$

Available since 2.0, vectorized in 2.13

R atan2(T y, T x)

Return the principal arc (inverse) tangent (in radians) of y divided by x,

$$\text{atan2}(y, x) = \arctan\left(\frac{y}{x}\right)$$

Available since 2.0, vectorized in 2.34

3.10. Hyperbolic trigonometric functions

R cosh(T x)

The hyperbolic cosine of x (in radians)

Available since 2.0, vectorized in 2.13

R sinh(T x)

The hyperbolic sine of x (in radians)

Available since 2.0, vectorized in 2.13

R tanh(T x)

The hyperbolic tangent of x (in radians)

Available since 2.0, vectorized in 2.13

R acosh(T x)

The inverse hyperbolic cosine (in radians)

Available since 2.0, vectorized in 2.13

R asinh(T x)

The inverse hyperbolic sine (in radians)

Available since 2.0, vectorized in 2.13

R atanh(T x)

The inverse hyperbolic tangent (in radians) of x

Available since 2.0, vectorized in 2.13

3.11. Link functions

The following functions are commonly used as link functions in generalized linear models. The function Φ is also commonly used as a link function (see section [probability-related functions](#)).

R logit(T x)

The log odds, or logit, function applied to x

Available since 2.0, vectorized in 2.13

R `inv_logit`(T x)

The logistic sigmoid function applied to x

Available since 2.0, vectorized in 2.13

R `inv_cloglog`(T x)

The inverse of the complementary log-log function applied to x

Available since 2.0, vectorized in 2.13

3.12. Probability-related functions

Normal cumulative distribution functions

The error function erf is related to the standard normal cumulative distribution function Φ by scaling. See section [normal distribution](#) for the general normal cumulative distribution function (and its complement).

R `erf`(T x)

The error function, also known as the Gauss error function, of x

Available since 2.0, vectorized in 2.13

R `erfc`(T x)

The complementary error function of x

Available since 2.0, vectorized in 2.13

R `inv_erfc`(T x)

The inverse of the complementary error function of x

Available since 2.29, vectorized in 2.29

R `Phi`(T x)

The standard normal cumulative distribution function of x

Available since 2.0, vectorized in 2.13

R `inv_Phi`(T x)

Return the value of the inverse standard normal cdf Φ^{-1} at the specified quantile x . The details of the algorithm can be found in (Wichura 1988). Quantile arguments below 1e-16 are untested; quantiles above 0.99999999 result in increasingly large errors.

Available since 2.0, vectorized in 2.13

R Phi_approx(T x)

The fast approximation of the unit (may replace Phi for probit regression with maximum absolute error of 0.00014, see (Bowling et al. 2009) for details)

Available since 2.0, vectorized in 2.13

Other probability-related functions**real binary_log_loss(int y, real y_hat)**

Return the log loss function for predicting $\hat{y} \in [0,1]$ for boolean outcome $y \in \{0,1\}$.

$$\text{binary_log_loss}(y, \hat{y}) = \begin{cases} -\log \hat{y} & \text{if } y = 1 \\ -\log(1 - \hat{y}) & \text{otherwise} \end{cases}$$

Available since 2.0

R binary_log_loss(T1 x, T2 y)

Vectorized implementation of the `binary_log_loss` function

Available since 2.25

real owens_t(real h, real a)

Return the Owen's T function for the probability of the event $X > h$ and $0 < Y < aX$ where X and Y are independent standard normal random variables.

$$\text{owens_t}(h, a) = \frac{1}{2\pi} \int_0^a \frac{\exp(-\frac{1}{2}h^2(1+x^2))}{1+x^2} dx$$

Available since 2.25

R owens_t(T1 x, T2 y)

Vectorized implementation of the `owens_t` function

Available since 2.25

3.13. Combinatorial functions**real beta(real alpha, real beta)**

Return the beta function applied to alpha and beta. The beta function, $B(\alpha, \beta)$, computes the normalizing constant for the beta distribution, and is defined for $\alpha > 0$ and $\beta > 0$. See section [appendix](#) for definition of $B(\alpha, \beta)$.

Available since 2.25

R beta(T1 x, T2 y)

Vectorized implementation of the `beta` function

Available since 2.25

real inc_beta(real alpha, real beta, real x)

Return the regularized incomplete beta function up to x applied to alpha and beta. See section [appendix](#) for a definition.

Available since 2.10

real inv_inc_beta(real alpha, real beta, real p)

Return the inverse of the regularized incomplete beta function. The return value x is the value that solves p = inc_beta(alpha, beta, x). See section [appendix](#) for a definition of the inc_beta.

Available since 2.30

real lbeta(real alpha, real beta)

Return the natural logarithm of the beta function applied to alpha and beta. The beta function, $B(\alpha, \beta)$, computes the normalizing constant for the beta distribution, and is defined for $\alpha > 0$ and $\beta > 0$.

$$\text{lbeta}(\alpha, \beta) = \log \Gamma(\alpha) + \log \Gamma(\beta) - \log \Gamma(\alpha + \beta)$$

See section [appendix](#) for definition of $B(\alpha, \beta)$.

Available since 2.0

R lbeta(T1 x, T2 y)

Vectorized implementation of the lbeta function

Available since 2.25

R tgamma(T x)

The gamma function applied to x. The gamma function is the generalization of the factorial function to continuous variables, defined so that $\Gamma(n + 1) = n!$. See for a full definition of $\Gamma(x)$. The function is defined for positive numbers and non-integral negative numbers,

Available since 2.0, vectorized in 2.13

R lgamma(T x)

The natural logarithm of the gamma function applied to x,

Available since 2.0, vectorized in 2.15

R digamma(T x)

The digamma function applied to x. The digamma function is the derivative of

the natural logarithm of the Gamma function. The function is defined for positive numbers and non-integral negative numbers

Available since 2.0, vectorized in 2.13

R `trigamma`(T x)

The trigamma function applied to x. The trigamma function is the second derivative of the natural logarithm of the Gamma function

Available since 2.0, vectorized in 2.13

real `lmgamma`(int n, real x)

Return the natural logarithm of the multivariate gamma function Γ_n with n dimensions applied to x.

$$\text{lmgamma}(n, x) = \begin{cases} \frac{n(n-1)}{4} \log \pi + \sum_{j=1}^n \log \Gamma\left(x + \frac{1-j}{2}\right) & \text{if } x \notin \{\dots, -3, -2, -1, 0\} \\ \text{error} & \text{otherwise} \end{cases}$$

Available since 2.0

R `lmgamma`(T1 x, T2 y)

Vectorized implementation of the `lmgamma` function

Available since 2.25

real `gamma_p`(real a, real z)

Return the normalized lower incomplete gamma function of a and z defined for positive a and nonnegative z.

$$\text{gamma_p}(a, z) = \begin{cases} \frac{1}{\Gamma(a)} \int_0^z t^{a-1} e^{-t} dt & \text{if } a > 0, z \geq 0 \\ \text{error} & \text{otherwise} \end{cases}$$

Available since 2.0

R `gamma_p`(T1 x, T2 y)

Vectorized implementation of the `gamma_p` function

Available since 2.25

real `gamma_q`(real a, real z)

Return the normalized upper incomplete gamma function of a and z defined for

positive a and nonnegative z.

$$\text{gamma_q}(a, z) = \begin{cases} \frac{1}{\Gamma(a)} \int_z^\infty t^{a-1} e^{-t} dt & \text{if } a > 0, z \geq 0 \\ \text{error} & \text{otherwise} \end{cases}$$

Available since 2.0

R **gamma_q**(T1 x, T2 y)

Vectorized implementation of the gamma_q function

Available since 2.25

int choose(int x, int y)

Return the binomial coefficient of x and y. For non-negative integer inputs, the binomial coefficient function is written as $\binom{x}{y}$ and pronounced “x choose y.” In its the antilog of the lchoose function but returns an integer rather than a real number with no non-zero decimal places. For $0 \leq y \leq x$, the binomial coefficient function can be defined via the factorial function

$$\text{choose}(x, y) = \frac{x!}{(y!) (x - y)!}.$$

Available since 2.14

R **choose**(T1 x, T2 y)

Vectorized implementation of the choose function

Available since 2.25

real bessel_first_kind(int v, real x)

Return the Bessel function of the first kind with order v applied to x.

$$\text{bessel_first_kind}(v, x) = J_v(x),$$

where

$$J_v(x) = \left(\frac{1}{2}x\right)^v \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{4}x^2\right)^k}{k! \Gamma(v + k + 1)}$$

Available since 2.5

R **bessel_first_kind**(T1 x, T2 y)

Vectorized implementation of the bessel_first_kind function

Available since 2.25

real bessel_second_kind(int v, real x)

Return the Bessel function of the second kind with order v applied to x defined for positive x and v. For $x, v > 0$,

$$\text{bessel_second_kind}(v, x) = \begin{cases} Y_v(x) & \text{if } x > 0 \\ \text{error} & \text{otherwise} \end{cases}$$

where

$$Y_v(x) = \frac{J_v(x) \cos(v\pi) - J_{-v}(x)}{\sin(v\pi)}$$

Available since 2.5

R bessel_second_kind(T1 x, T2 y)

Vectorized implementation of the bessel_second_kind function

Available since 2.25

real modified_bessel_first_kind(int v, real z)

Return the modified Bessel function of the first kind with order v applied to z defined for all z and integer v.

$$\text{modified_bessel_first_kind}(v, z) = I_v(z)$$

where

$$I_v(z) = \left(\frac{1}{2}z\right)^v \sum_{k=0}^{\infty} \frac{\left(\frac{1}{4}z^2\right)^k}{k!\Gamma(v+k+1)}$$

Available since 2.1

R modified_bessel_first_kind(T1 x, T2 y)

Vectorized implementation of the modified_bessel_first_kind function

Available since 2.25

real log_modified_bessel_first_kind(real v, real z)

Return the log of the modified Bessel function of the first kind. v does not have to be an integer.

Available since 2.26

R log_modified_bessel_first_kind(T1 x, T2 y)

Vectorized implementation of the log_modified_bessel_first_kind function

Available since 2.26

real modified_bessel_second_kind(int v, real z)

Return the modified Bessel function of the second kind with order v applied to z defined for positive z and integer v.

$$\text{modified_bessel_second_kind}(v, z) = \begin{cases} K_v(z) & \text{if } z > 0 \\ \text{error} & \text{if } z \leq 0 \end{cases}$$

where

$$K_v(z) = \frac{\pi}{2} \cdot \frac{I_{-v}(z) - I_v(z)}{\sin(v\pi)}$$

Available since 2.1

R modified_bessel_second_kind(T1 x, T2 y)

Vectorized implementation of the `modified_bessel_second_kind` function

Available since 2.25

real falling_factorial(real x, real n)

Return the falling factorial of x with power n defined for positive x and real n.

$$\text{falling_factorial}(x, n) = \begin{cases} (x)_n & \text{if } x > 0 \\ \text{error} & \text{if } x \leq 0 \end{cases}$$

where

$$(x)_n = \frac{\Gamma(x + 1)}{\Gamma(x - n + 1)}$$

Available since 2.0

R falling_factorial(T1 x, T2 y)

Vectorized implementation of the `falling_factorial` function

Available since 2.25

real lchoose(real x, real y)

Return the natural logarithm of the generalized binomial coefficient of x and y. For non-negative integer inputs, the binomial coefficient function is written as $\binom{x}{y}$ and pronounced “x choose y.” This function generalizes to real numbers using the gamma function. For $0 \leq y \leq x$,

$$\text{binomial_coefficient_log}(x, y) = \log \Gamma(x + 1) - \log \Gamma(y + 1) - \log \Gamma(x - y + 1).$$

Available since 2.10

R lchoose(T1 x, T2 y)

Vectorized implementation of the `lchoose` function

Available since 2.29

real log_falling_factorial(real x, real n)

Return the log of the falling factorial of x with power n defined for positive x and real n.

$$\log_{\text{falling}}_{\text{factorial}}(x, n) = \begin{cases} \log(x)_n & \text{if } x > 0 \\ \text{error} & \text{if } x \leq 0 \end{cases}$$

Available since 2.0

real rising_factorial(real x, int n)

Return the rising factorial of x with power n defined for positive x and integer n.

$$\text{rising}_{\text{factorial}}(x, n) = \begin{cases} x^{(n)} & \text{if } x > 0 \\ \text{error} & \text{if } x \leq 0 \end{cases}$$

where

$$x^{(n)} = \frac{\Gamma(x + n)}{\Gamma(x)}$$

Available since 2.20

R rising_factorial(T1 x, T2 y)

Vectorized implementation of the `rising_factorial` function

Available since 2.25

real log_rising_factorial(real x, real n)

Return the log of the rising factorial of x with power n defined for positive x and real n.

$$\log_{\text{rising}}_{\text{factorial}}(x, n) = \begin{cases} \log x^{(n)} & \text{if } x > 0 \\ \text{error} & \text{if } x \leq 0 \end{cases}$$

Available since 2.0

R log_rising_factorial(T1 x, T2 y)

Vectorized implementation of the `log_rising_factorial` function

Available since 2.25

3.14. Composed functions

The functions in this section are equivalent in theory to combinations of other functions. In practice, they are implemented to be more efficient and more numerically stable than defining them directly using more basic Stan functions.

R **expm1**(T x)

The natural exponential of x minus 1

Available since 2.0, vectorized in 2.13

real **fma**(real x, real y, real z)

Return z plus the result of x multiplied by y.

$$\text{fma}(x, y, z) = (x \times y) + z$$

Available since 2.0

real **ldexp**(real x, int y)

Return the product of x and two raised to the y power.

$$\text{ldexp}(x, y) = x2^y$$

Available since 2.25

R **ldexp**(T1 x, T2 y)

Vectorized implementation of the **ldexp** function

Available since 2.25

real **lmultiply**(real x, real y)

Return the product of x and the natural logarithm of y.

$$\text{lmultiply}(x, y) = \begin{cases} 0 & \text{if } x = y = 0 \\ x \log y & \text{if } x, y \neq 0 \\ \text{NaN} & \text{otherwise} \end{cases}$$

Available since 2.10

R **lmultiply**(T1 x, T2 y)

Vectorized implementation of the **lmultiply** function

Available since 2.25

R `log1p`(T x)

The natural logarithm of 1 plus x

Available since 2.0, vectorized in 2.13

R `log1m`(T x)

The natural logarithm of 1 minus x

Available since 2.0, vectorized in 2.13

R `log1p_exp`(T x)

The natural logarithm of one plus the natural exponentiation of x

Available since 2.0, vectorized in 2.13

R `log1m_exp`(T x)

The logarithm of one minus the natural exponentiation of x

Available since 2.0, vectorized in 2.13

real `log_diff_exp`(real x , real y)

Return the natural logarithm of the difference of the natural exponentiation of x and the natural exponentiation of y .

$$\log_diff_exp(x, y) = \begin{cases} \log(\exp(x) - \exp(y)) & \text{if } +\infty > x \geq y \\ \text{NaN} & \text{otherwise} \end{cases}$$

When x is equal to y , `log_diff_exp(x, y)` returns $-\infty$, consistent with `log(0)` returning $-\infty$. This includes the case in which x and y are both equal to $-\infty$, which corresponds to `log(0 - 0)` because `exp(negative_infinity())` returns 0.

Available since 2.0

R `log_diff_exp`($T1$ x , $T2$ y)

Vectorized implementation of the `log_diff_exp` function

Available since 2.25

real `log_mix`(real θ , real $lp1$, real $lp2$)

Return the log mixture of the log densities $lp1$ and $lp2$ with mixing proportion θ , defined by

$$\begin{aligned} \log_mix(\theta, \lambda_1, \lambda_2) &= \log(\theta \exp(\lambda_1) + (1 - \theta) \exp(\lambda_2)) \\ &= \log_sum_exp(\log(\theta) + \lambda_1, \log(1 - \theta) + \lambda_2). \end{aligned}$$

Available since 2.6

R **log_mix**(T1 thetas, T2 lps)

Calculates the log mixture density given `thetas`, mixing proportions which should be between 0 and 1 and sum to 1, and `lps`, log densities. The `lps` variable must be either a 1-d container of the same length as `thetas`, or an array of such.

$$\begin{aligned}\log_mix(\theta, \lambda) &= \log\left(\sum_{n=1}^N \theta_n * \exp(\lambda_n)\right) \\ &= \log_sum_exp(\log(\theta) + \lambda).\end{aligned}$$

This is a generalization of the above signature of three arguments to more than two densities. For example, `log_mix(lambda, lp1, lp2) == log_mix({lambda, 1 - lambda}, {lp1, lp2})`.

Available since 2.26

R **log_sum_exp**(T1 x, T2 y)

Return the natural logarithm of the sum of the natural exponentiation of `x` and the natural exponentiation of `y`.

$$\log_sum_exp(x, y) = \log(\exp(x) + \exp(y))$$

Available since 2.0, vectorized in 2.33

R **log_inv_logit**(T x)

The natural logarithm of the inverse logit function of `x`

Available since 2.0, vectorized in 2.13

R **log_inv_logit_diff**(T1 x, T2 y)

The natural logarithm of the difference of the inverse logit function of `x` and the inverse logit function of `y`

Available since 2.25

R **log1m_inv_logit**(T x)

The natural logarithm of 1 minus the inverse logit function of `x`

Available since 2.0, vectorized in 2.13

3.15. Special functions

R `lambert_w0(T x)`

Implementation of the W_0 branch of the Lambert W function, i.e., solution to the function $W_0(x) \exp^{W_0(x)} = x$

Available since 2.25

R `lambert_wm1(T x)`

Implementation of the W_{-1} branch of the Lambert W function, i.e., solution to the function $W_{-1}(x) \exp^{W_{-1}(x)} = x$

Available since 2.25

3.16. Hypergeometric Functions

Hypergeometric functions refer to a power series of the form

$${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n \cdots (a_p)_n}{(b_1)_n \cdots (b_q)_n} \frac{z^n}{n!}$$

where $(a)_n$ is the Pochhammer symbol defined as $(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)}$.

The gradients of the hypergeometric function are given by:

$$\begin{aligned} \frac{\partial}{\partial a_1} &= \sum_{k=0}^{\infty} \frac{\psi(k+a_1) \left(\prod_{j=1}^p (a_j)_k \right) z^k}{k! \prod_{j=1}^q (b_j)_k} - \psi(a_1) {}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z) \\ \frac{\partial}{\partial b_1} &= \psi(b_1) {}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z) - \sum_{k=0}^{\infty} \frac{\psi(k+b_1) \left(\prod_{j=1}^p (a_j)_k \right) z^k}{k! \prod_{j=1}^q (b_j)_k} \\ \frac{\partial}{\partial z} &= \frac{\prod_{j=1}^p a_j}{\prod_{j=1}^q b_j} {}_pF_q(a_1 + 1, \dots, a_p + 1; b_1 + 1, \dots, b_q + 1; z) \end{aligned}$$

Stan provides both the generalized hypergeometric function as well as several special cases for particular values of p and q .

real `hypergeometric_1F0(real a, real z)`

Special case of the hypergeometric function with $p = 1$ and $q = 0$.

Available since 2.37

real hypergeometric_2F1(real a1, real a2, real b1, real z)

Special case of the hypergeometric function with $p = 2$ and $q = 1$. If the function does not meet convergence criteria for given inputs, the function will attempt to apply [Euler's transformation](#) to improve convergence:

$${}_2F_1(a_1, a_2, b_1, z) = {}_2F_1(b_1 - a_1, a_2, b_1, \frac{z}{z-1}) \cdot (1-z)^{-a_2}$$

Available since 2.37

real hypergeometric_3F2(T1 a, T2 b, real z)

Special case of the hypergeometric function with $p = 3$ and $q = 2$, where a and b are vectors of length 3 and 2, respectively.

Available since 2.37

real hypergeometric_pFq(T1 a, T2 b, real z)

Generalized hypergeometric function, where a and b are vectors of length p and q, respectively.

Available since 2.37

4. Complex-Valued Basic Functions

This chapter describes built-in functions that operate on complex numbers, either as an argument type or a return type. This includes the arithmetic operators generalized to complex numbers.

4.1. Complex assignment and promotion

Just as integers may be assigned to real variables, real variables may be assigned to complex numbers, with the result being a zero imaginary component.

```
int n = 5;           // n = 5
real x = a;          // x = 5.0
complex z1 = n;     // z = 5.0 + 0.0i
complex z2 = x;     // z = 5.0 + 0.0i
```

Complex function arguments

Function arguments of type `int` or `real` may be promoted to type `complex`. The complex version of functions in this chapter are only used if one of the arguments is complex. For example, if `z` is complex, then `pow(z, 2)` will call the complex version of the power function and the integer 2 will be promoted to a complex number with a real component of 2 and an imaginary component of 0. The same goes for binary operators like addition and subtraction, where `z + 2` will be legal and produce a complex result. Functions such as `arg` and `conj` that are only available for complex numbers can accept integer or real arguments, promoting them to `complex` before applying the function.

4.2. Complex constructors and accessors

Complex constructors

Variables and constants of type `complex` are constructed from zero, one, or two real numbers.

```
complex z1 = to_complex();           // z1 = 0.0 + 0.0i
real re = -2.9;
complex z2 = to_complex(re);         // z2 = -2.9 + 0.0i
real im = 1.3;
complex z3 = to_complex(re, im);    // z3 = -2.9 + 1.3i
```

`complex to_complex()`

Return complex number with real part 0.0 and imaginary part 0.0.

Available since 2.28

`complex to_complex(real re)`

Return complex number with real part `re` and imaginary part 0.0.

Available since 2.28

`complex to_complex(real re, real im)`

Return complex number with real part `re` and imaginary part `im`.

Available since 2.28

`z to_complex(T1 re, T2 im)`

Vectorized implementation of the `to_complex` function.

`T1` and `T2` can either be real containers of the same size, or a real container and a real, in which case the real value is used for the corresponding component in all elements of the output.

Available since 2.30

Complex accessors

Given a complex number, its real and imaginary parts can be extracted with the following functions.

`real get_real(complex z)`

Return the real part of the complex number `z`.

Available since 2.28

`real get_imag(complex z)`

Return the imaginary part of the complex number `z`.

Available since 2.28

4.3. Complex arithmetic operators

The arithmetic operators have the same precedence for complex and real arguments. The complex form of an operator will be selected if at least one of its argument is of type `complex`. If there are two arguments and only one is of type `complex`, then the other will be promoted to type `complex` before performing the operation.

Unary operators**complex operator+(complex z)**Return the complex argument z ,

$$+z = z.$$

*Available since 2.28***complex operator-(complex z)**Return the negation of the complex argument z , which for $z = x + yi$ is

$$-z = -x - yi.$$

*Available since 2.28***T operator-(T x)**Vectorized version of **operator-**. If $T x$ is a (possibly nested) array of complex numbers, $-x$ is the same shape array where each individual value is negated.*Available since 2.31***Binary operators****complex operator+(complex x, complex y)**Return the sum of x and y ,

$$(x + y) = \text{operator+}(x, y) = x + y.$$

*Available since 2.28***complex operator-(complex x, complex y)**Return the difference between x and y ,

$$(x - y) = \text{operator-}(x, y) = x - y.$$

*Available since 2.28***complex operator*(complex x, complex y)**Return the product of x and y ,

$$(x * y) = \text{operator*}(x, y) = x \times y.$$

Available since 2.28

`complex operator/(complex x, complex y)`

Return the quotient of x and y,

$$(x/y) = \text{operator}/(x, y) = \frac{x}{y}$$

Available since 2.28

`complex operator^(complex x, complex y)`

Return x raised to the power of y,

$$(x^y) = \text{operator}^(x, y) = \exp(y \log(x)).$$

Available since 2.28

4.4. Complex comparison operators

Complex numbers are equal if and only if both their real and imaginary components are equal. That is, the conditional

`z1 == z2`

is equivalent to

`get_real(z1) == get_real(z2) && get_imag(z1) == get_imag(z2)`

As with other complex functions, if one of the arguments is of type `real` or `int`, it will be promoted to type `complex` before comparison. For example, if `z` is of type `complex`, then `z == 0` will be true if `z` has real component equal to 0.0 and complex component equal to 0.0.

Warning: As with real values, it is usually a mistake to compare complex numbers for equality because their parts are implemented using floating-point arithmetic, which suffers from precision errors, rendering algebraically equivalent expressions not equal after evaluation.

`int operator==(complex x, complex y)`

Return 1 if x is equal to y and 0 otherwise,

$$(x == y) = \text{operator}==(x, y) = \begin{cases} 1 & \text{if } x = y, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

Available since 2.28

`int operator!=(complex x, complex y)`
 Return 1 if x is not equal to y and 0 otherwise,

$$(x \neq y) = \text{operator} !=(x, y) = \begin{cases} 1 & \text{if } x \neq y, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

Available since 2.28

4.5. Complex (compound) assignment operators

The assignment operator only serves as a component in the assignment statement and is thus not technically a function in the Stan language. With that caveat, it is documented here for completeness.

Assignment of complex numbers works elementwise. If an expression of type `int` or `real` is assigned to a complex number, it will be promoted before assignment as if calling `to_complex()`, so that the imaginary component is 0.0.

`void operator=(complex x, complex y)`
`y = x;` assigns a (copy of) the value of y to x.

Available since 2.28

`void operator+=(complex x, complex y)`
`x += y;` is equivalent to `x = x + y;`.

Available since 2.28

`void operator-=(complex x, complex y)`
`x -= y;` is equivalent to `x = x - y;`.

Available since 2.28

`void operator*=(complex x, complex y)`
`x *= y;` is equivalent to `x = x * y;`.

Available since 2.28

`void operator/=(complex x, complex y)`
`x /= y;` is equivalent to `x = x / y;`.

Available since 2.28

4.6. Complex special functions

The following functions are specific to complex numbers other than absolute value, which has a specific meaning for complex numbers.

real abs(complex z)

Return the absolute value of z , also known as the modulus or magnitude, which for $z = x + yi$ is

$$\text{abs}(z) = \sqrt{x^2 + y^2}.$$

This function works elementwise over containers, returning the same shape and kind of the input container but holding reals. For example, a `complex_vector[n]` input will return a `vector[n]` output, with each element transformed by the above equation.

Available since 2.28, vectorized in 2.30

real arg(complex z)

Return the phase angle (in radians) of z , which for $z = x + yi$ is

$$\text{arg}(z) = \text{atan2}(y, x) = \text{atan}(y/x).$$

Available since 2.28

real norm(complex z)

Return the Euclidean norm of z , which is its absolute value squared, and which for $z = x + yi$ is

$$\text{norm}(z) = \text{abs}^2(z) = x^2 + y^2.$$

Available since 2.28

complex conj(complex z)

Return the complex conjugate of z , which negates the imaginary component, so that if $z = x + yi$,

$$\text{conj}(z) = x - yi.$$

Available since 2.28

 $\mathbf{z}\ conj(z)$

Vectorized version of `conj`. This will apply the `conj` function to each element of a complex array, vector, or matrix.

Available since 2.31

complex proj(complex z)

Return the projection of z onto the Riemann sphere, which for $z = x + yi$ is

$$\text{proj}(z) = \begin{cases} z & \text{if } z \text{ is finite, and} \\ 0 + \text{sign}(y)i & \text{otherwise,} \end{cases}$$

where $\text{sign}(y)$ is -1 if y is negative and 1 otherwise.

Available since 2.28

complex polar(real r , real θ)

Return the complex number with magnitude (absolute value) r and phase angle θ .

Available since 2.28

4.7. Complex exponential and power functions

The exponential, log, and power functions may be supplied with complex arguments with specialized meanings that generalize their real counterparts. These versions are only called when the argument is complex.

complex exp(complex z)

Return the complex natural exponential of z , which for $z = x + yi$ is

$$\exp z = \exp(x)\text{cis}(y) = \exp(x)(\cos(y) + i \sin(y)).$$

Available since 2.28

complex log(complex z)

Return the complex natural logarithm of z , which for $z = \text{polar}(r, \theta)$ is

$$\log z = \log r + \theta i.$$

Available since 2.28

complex log10(complex z)

Return the complex common logarithm of z ,

$$\log_{10} z = \frac{\log z}{\log 10}.$$

Available since 2.28

complex pow(complex x , complex y)

Return x raised to the power of y ,

$$\text{pow}(x, y) = \exp(y \log(x)).$$

Available since 2.28

`z pow(T1 x, T2 y)`

Vectorized implementation of the `pow` function

Available since 2.30

`complex sqrt(complex x)`

Return the complex square root of `x` with branch cut along the negative real axis. For finite inputs, the result will be in the right half-plane.

Available since 2.28

4.8. Complex trigonometric functions

The standard trigonometric functions are supported for complex numbers.

`complex cos(complex z)`

Return the complex cosine of `z`, which is

$$\cos(z) = \cosh(z i) = \frac{\exp(z i) + \exp(-z i)}{2}.$$

Available since 2.28

`complex sin(complex z)`

Return the complex sine of `z`,

$$\sin(z) = -\sinh(z i) i = \frac{\exp(z i) - \exp(-z i)}{2 i}.$$

Available since 2.28

`complex tan(complex z)`

Return the complex tangent of `z`,

$$\tan(z) = -\tanh(z i) i = \frac{(\exp(-z i) - \exp(z i)) i}{\exp(-z i) + \exp(z i)}.$$

Available since 2.28

`complex acos(complex z)`

Return the complex arc (inverse) cosine of `z`,

$$\text{acos}(z) = \frac{1}{2}\pi + \log(z i + \sqrt{1 - z^2}) i.$$

Available since 2.28

complex asin(complex z)

Return the complex arc (inverse) sine of z,

$$\text{asin}(z) = -\log(z i + \sqrt{1 - z^2}) i.$$

Available since 2.28

complex atan(complex z)

Return the complex arc (inverse) tangent of z,

$$\text{atan}(z) = -\frac{1}{2}(\log(1 - z i) - \log(1 + z i)) i.$$

Available since 2.28

4.9. Complex hyperbolic trigonometric functions

The standard hyperbolic trigonometric functions are supported for complex numbers.

complex cosh(complex z)

Return the complex hyperbolic cosine of z,

$$\cosh(z) = \frac{\exp(z) + \exp(-z)}{2}.$$

Available since 2.28

complex sinh(complex z)

Return the complex hyperbolic sine of z,

$$\sinh(z) = \frac{\exp(z) - \exp(-z)}{2}.$$

Available since 2.28

complex tanh(complex z)

Return the complex hyperbolic tangent of z,

$$\tanh(z) = \frac{\sinh(z)}{\cosh(z)} = \frac{\exp(z) - \exp(-z)}{\exp(z) + \exp(-z)}.$$

Available since 2.28

`complex acosh(complex z)`

Return the complex hyperbolic arc (inverse) cosine of z ,

$$\operatorname{acosh}(z) = \log(z + \sqrt{(z + 1)(z - 1)}).$$

Available since 2.28

`complex asinh(complex z)`

Return the complex hyperbolic arc (inverse) sine of z ,

$$\operatorname{asinh}(z) = \log(z + \sqrt{1 + z^2}).$$

Available since 2.28

`complex atanh(complex z)`

Return the complex hyperbolic arc (inverse) tangent of z ,

$$\operatorname{atanh}(z) = \frac{\log(1 + z) - \log(1 - z)}{2}.$$

Available since 2.28

5. Array Operations

5.1. Reductions

The following operations take arrays as input and produce single output values. The boundary values for size 0 arrays are the unit with respect to the combination operation (min, max, sum, or product).

Minimum and maximum

`real min(array[] real x)`

The minimum value in `x`, or $+\infty$ if `x` is size 0.

Available since 2.0

`int min(array[] int x)`

The minimum value in `x`, or error if `x` is size 0.

Available since 2.0

`real max(array[] real x)`

The maximum value in `x`, or $-\infty$ if `x` is size 0.

Available since 2.0

`int max(array[] int x)`

The maximum value in `x`, or error if `x` is size 0.

Available since 2.0

Sum, product, and log sum of exp

`int sum(array[] int x)`

The sum of the elements in `x`, or 0 if the array is empty.

Available since 2.1

`real sum(array[] real x)`

The sum of the elements in `x`; see definition above.

Available since 2.0

`complex sum(array[] complex x)`

The sum of the elements in `x`; see definition above.

Available since 2.30

real prod(array[] real x)

The product of the elements in x , or 1 if x is size 0.

Available since 2.0

real prod(array[] int x)

The product of the elements in x ,

$$\text{product}(x) = \begin{cases} \prod_{n=1}^N x_n & \text{if } N > 0 \\ 1 & \text{if } N = 0 \end{cases}$$

Available since 2.0

real log_sum_exp(array[] real x)

The natural logarithm of the sum of the exponentials of the elements in x , or $-\infty$ if the array is empty.

Available since 2.0

Sample mean, variance, and standard deviation

The sample mean, variance, and standard deviation are calculated in the usual way. For i.i.d. draws from a distribution of finite mean, the sample mean is an unbiased estimate of the mean of the distribution. Similarly, for i.i.d. draws from a distribution of finite variance, the sample variance is an unbiased estimate of the variance.¹ The sample deviation is defined as the square root of the sample deviation, but is not unbiased.

real mean(array[] real x)

The sample mean of the elements in x . For an array x of size $N > 0$,

$$\text{mean}(x) = \bar{x} = \frac{1}{N} \sum_{n=1}^N x_n.$$

It is an error to the call the mean function with an array of size 0.

Available since 2.0

¹Dividing by N rather than $(N - 1)$ produces a maximum likelihood estimate of variance, which is biased to underestimate variance.

real variance(array[] real x)

The sample variance of the elements in x. For $N > 0$,

$$\text{variance}(x) = \begin{cases} \frac{1}{N-1} \sum_{n=1}^N (x_n - \bar{x})^2 & \text{if } N > 1 \\ 0 & \text{if } N = 1 \end{cases}$$

It is an error to call the variance function with an array of size 0.

Available since 2.0

real sd(array[] real x)

The sample standard deviation of elements in x.

$$\text{sd}(x) = \begin{cases} \sqrt{\text{variance}(x)} & \text{if } N > 1 \\ 0 & \text{if } N = 0 \end{cases}$$

It is an error to call the sd function with an array of size 0.

Available since 2.0

Norms

real norm1(vector x)

The L1 norm of x, defined by

$$\text{norm1}(x) = \sum_{n=1}^N |x_n|$$

where N is the size of x.

Available since 2.30

real norm1(row_vector x)

The L1 norm of x

Available since 2.30

real norm1(array[] real x)

The L1 norm of x

Available since 2.30

real norm2(vector x)

The L2 norm of x, defined by

$$\text{norm2}(x) = \sqrt{\sum_{n=1}^N (x_n)^2}$$

where N is the size of x

Available since 2.30

real norm2(row_vector x)

The L2 norm of x

Available since 2.30

real norm2(array[] real x)

The L2 norm of x

Available since 2.30

Euclidean distance and squared distance

real distance(vector x, vector y)

The Euclidean distance between x and y , defined by

$$\text{distance}(x, y) = \sqrt{\sum_{n=1}^N (x_n - y_n)^2}$$

where N is the size of x and y . It is an error to call `distance` with arguments of unequal size.

Available since 2.2

real distance(vector x, row_vector y)

The Euclidean distance between x and y

Available since 2.2

real distance(row_vector x, vector y)

The Euclidean distance between x and y

Available since 2.2

real distance(row_vector x, row_vector y)

The Euclidean distance between x and y

Available since 2.2

real squared_distance(vector x, vector y)

The squared Euclidean distance between x and y , defined by

$$\text{quared_distance}(x, y) = \text{distance}(x, y)^2 = \sum_{n=1}^N (x_n - y_n)^2,$$

where N is the size of x and y . It is an error to call `squared_distance` with arguments of unequal size.

Available since 2.7

`real squared_distance(vector x, row_vector y)`

The squared Euclidean distance between x and y

Available since 2.26

`real squared_distance(row_vector x, vector y)`

The squared Euclidean distance between x and y

Available since 2.26

`real squared_distance(row_vector x, row_vector y)`

The Euclidean distance between x and y

Available since 2.26

Quantile

Produces sample quantiles corresponding to the given probabilities. The smallest observation corresponds to a probability of 0 and the largest to a probability of 1.

Implements algorithm 7 from Hyndman, R. J. and Fan, Y., Sample quantiles in Statistical Packages (R's default quantile function).

`real quantile(data array[] real x, data real p)`

The p-th quantile of x

Available since 2.27

`array[] real quantile(data array[] real x, data array[] real p)`

An array containing the quantiles of x given by the array of probabilities p

Available since 2.27

5.2. Array size and dimension function

The size of an array or matrix can be obtained using the `dims()` function. The `dims()` function is defined to take an argument consisting of any variable with up to 8 array dimensions (and up to 2 additional matrix dimensions) and returns an array of integers with the dimensions. For example, if two variables are declared as follows,

```
array[7, 8, 9] real x;
array[7] matrix[8, 9] y;
```

then calling `dims(x)` or `dims(y)` returns an integer array of size 3 containing the elements 7, 8, and 9 in that order.

The `size()` function extracts the number of elements in an array. This is just the top-level elements, so if the array is declared as

```
array[M, N] real a;
```

the size of `a` is `M`.

The function `num_elements`, on the other hand, measures all of the elements, so that the array `a` above has $M \times N$ elements.

The specialized functions `rows()` and `cols()` should be used to extract the dimensions of vectors and matrices.

```
array[] int dims(T x)
```

Return an integer array containing the dimensions of `x`; the type of the argument `T` can be any Stan type with up to 8 array dimensions.

Available since 2.0

```
int num_elements(array[] T x)
```

Return the total number of elements in the array `x` including all elements in contained arrays, vectors, and matrices. `T` can be any array type. For example, if `x` is of type `array[4, 3] real` then `num_elements(x)` is 12, and if `y` is declared as `array[5] matrix[3, 4] y`, then `size(y)` evaluates to 60.

Available since 2.5

```
int size(array[] T x)
```

Return the number of elements in the array `x`; the type of the array `T` can be any type, but the `size` is just the size of the top level array, not the total number of elements contained. For example, if `x` is of type `array[4, 3] real` then `size(x)` is 4.

Available since 2.0

5.3. Array broadcasting

The following operations create arrays by repeating elements to fill an array of a specified size. These operations work for all input types `T`, including reals, integers, vectors, row vectors, matrices, or arrays.

```
array[] T rep_array(T x, int n)
```

Return the `n` array with every entry assigned to `x`.

Available since 2.0

```
array [ , ] T rep_array(T x, int m, int n)
```

Return the m by n array with every entry assigned to x.

Available since 2.0

```
array [ , ] T rep_array(T x, int k, int m, int n)
```

Return the k by m by n array with every entry assigned to x.

Available since 2.0

For example, `rep_array(1.0,5)` produces a real array (type `array[] real`) of size 5 with all values set to 1.0. On the other hand, `rep_array(1,5)` produces an integer array (type `array[] int`) of size 5 with all values set to 1. This distinction is important because it is not possible to assign an integer array to a real array. For example, the following example contrasts legal with illegal array creation and assignment

```
array[5] real y;
array[5] int x;

x = rep_array(1, 5);      // ok
y = rep_array(1.0, 5);    // ok

x = rep_array(1.0, 5);    // illegal
y = rep_array(1, 5);      // illegal

x = y;                   // illegal
y = x;                   // illegal
```

If the value being repeated v is a vector (i.e., T is `vector`), then `rep_array(v, 27)` is a size 27 array consisting of 27 copies of the vector v.

```
vector[5] v;
array[3] vector[5] a;

a = rep_array(v, 3); // fill a with copies of v
a[2, 4] = 9.0;       // v[4], a[1, 4], a[3, 4] unchanged
```

If the type T of x is itself an array type, then the result will be an array with one, two, or three added dimensions, depending on which of the `rep_array` functions is called. For instance, consider the following legal code snippet.

```

array[5, 6] real a;
array[3, 4, 5, 6] real b;

b = rep_array(a, 3, 4); // make (3 x 4) copies of a
b[1, 1, 1, 1] = 27.9; // a[1, 1] unchanged

```

After the assignment to `b`, the value for `b[j, k, m, n]` is equal to `a[m, n]` where it is defined, for `j` in `1:3`, `k` in `1:4`, `m` in `1:5`, and `n` in `1:6`.

5.4. Array concatenation

`T append_array(T x, T y)`

Return the concatenation of two arrays in the order of the arguments. `T` must be an `N`-dimensional array of any Stan type (with a maximum `N` of 7). All dimensions but the first must match.

Available since 2.18

For example, the following code appends two three dimensional arrays of matrices together. Note that all dimensions except the first match. Any mismatches will cause an error to be thrown.

```

array[2, 1, 7] matrix[4, 6] x1;
array[3, 1, 7] matrix[4, 6] x2;
array[5, 1, 7] matrix[4, 6] x3;

x3 = append_array(x1, x2);

```

5.5. Sorting functions

Sorting can be used to sort values or the indices of those values in either ascending or descending order. For example, if `v` is declared as a real array of size 3, with values

$$v = (1, -10.3, 20.987),$$

then the various sort routines produce

$$\begin{aligned}
\text{sort_asc}(v) &= (-10.3, 1, 20.987) \\
\text{sort_desc}(v) &= (20.987, 1, -10.3) \\
\text{sort_indices_asc}(v) &= (2, 1, 3) \\
\text{sort_indices_desc}(v) &= (3, 1, 2)
\end{aligned}$$

array[] real **sort_asc**(array[] real v)

Sort the elements of v in ascending order

Available since 2.0

array[] int **sort_asc**(array[] int v)

Sort the elements of v in ascending order

Available since 2.0

array[] real **sort_desc**(array[] real v)

Sort the elements of v in descending order

Available since 2.0

array[] int **sort_desc**(array[] int v)

Sort the elements of v in descending order

Available since 2.0

array[] int **sort_indices_asc**(array[] real v)

Return an array of indices between 1 and the size of v, sorted to index v in ascending order.

Available since 2.3

array[] int **sort_indices_asc**(array[] int v)

Return an array of indices between 1 and the size of v, sorted to index v in ascending order.

Available since 2.3

array[] int **sort_indices_desc**(array[] real v)

Return an array of indices between 1 and the size of v, sorted to index v in descending order.

Available since 2.3

array[] int **sort_indices_desc**(array[] int v)

Return an array of indices between 1 and the size of v, sorted to index v in descending order.

Available since 2.3

int **rank**(array[] real v, int s)

Number of components of v less than v[s]

Available since 2.0

```
int rank(array[] int v, int s)
Number of components of v less than v[s]
```

Available since 2.0

5.6. Reversing functions

Stan provides functions to create a new array by reversing the order of elements in an existing array. For example, if v is declared as a real array of size 3, with values

$$v = (1, -10.3, 20.987),$$

then

$$\text{reverse}(v) = (20.987, -10.3, 1).$$

```
array[] T reverse(array[] T v)
```

Return a new array containing the elements of the argument in reverse order.

Available since 2.23

6. Matrix Operations

6.1. Integer-valued matrix size functions

`int num_elements(vector x)`

The total number of elements in the vector x (same as function `rows`)

Available since 2.5

`int num_elements(row_vector x)`

The total number of elements in the vector x (same as function `cols`)

Available since 2.5

`int num_elements(matrix x)`

The total number of elements in the matrix x. For example, if x is a 5×3 matrix, then `num_elements(x)` is 15

Available since 2.5

`int rows(vector x)`

The number of rows in the vector x

Available since 2.0

`int rows(row_vector x)`

The number of rows in the row vector x, namely 1

Available since 2.0

`int rows(matrix x)`

The number of rows in the matrix x

Available since 2.0

`int cols(vector x)`

The number of columns in the vector x, namely 1

Available since 2.0

`int cols(row_vector x)`

The number of columns in the row vector x

Available since 2.0

```
int cols(matrix x)
```

The number of columns in the matrix x

Available since 2.0

```
int size(vector x)
```

The size of x, i.e., the number of elements

Available since 2.26

```
int size(row_vector x)
```

The size of x, i.e., the number of elements

Available since 2.26

```
int size(matrix x)
```

The size of the matrix x. For example, if x is a 5×3 matrix, then `size(x)` is 15

Available since 2.26

6.2. Matrix arithmetic operators

Stan supports the basic matrix operations using infix, prefix and postfix operations. This section lists the operations supported by Stan along with their argument and result types.

Negation prefix operators

```
vector operator-(vector x)
```

The negation of the vector x.

Available since 2.0

```
row_vector operator-(row_vector x)
```

The negation of the row vector x.

Available since 2.0

```
matrix operator-(matrix x)
```

The negation of the matrix x.

Available since 2.0

```
T operator-(T x)
```

Vectorized version of `operator-`. If T x is a (possibly nested) array of matrix types, `-x` is the same shape array where each individual value is negated.

Available since 2.31

Infix matrix operators

`vector operator+(vector x, vector y)`
The sum of the vectors x and y.

Available since 2.0

`row_vector operator+(row_vector x, row_vector y)`
The sum of the row vectors x and y.

Available since 2.0

`matrix operator+(matrix x, matrix y)`
The sum of the matrices x and y

Available since 2.0

`vector operator-(vector x, vector y)`
The difference between the vectors x and y.

Available since 2.0

`row_vector operator-(row_vector x, row_vector y)`
The difference between the row vectors x and y

Available since 2.0

`matrix operator-(matrix x, matrix y)`
The difference between the matrices x and y

Available since 2.0

`vector operator*(real x, vector y)`
The product of the scalar x and vector y

Available since 2.0

`row_vector operator*(real x, row_vector y)`
The product of the scalar x and the row vector y

Available since 2.0

`matrix operator*(real x, matrix y)`
The product of the scalar x and the matrix y

Available since 2.0

`vector operator*(vector x, real y)`
The product of the scalar y and vector x

Available since 2.0

`matrix operator*(vector x, row_vector y)`

The product of the vector x and row vector y

Available since 2.0

`row_vector operator*(row_vector x, real y)`

The product of the scalar y and row vector x

Available since 2.0

`real operator*(row_vector x, vector y)`

The product of the row vector x and vector y

Available since 2.0

`row_vector operator*(row_vector x, matrix y)`

The product of the row vector x and matrix y

Available since 2.0

`matrix operator*(matrix x, real y)`

The product of the scalar y and matrix x

Available since 2.0

`vector operator*(matrix x, vector y)`

The product of the matrix x and vector y

Available since 2.0

`matrix operator*(matrix x, matrix y)`

The product of the matrices x and y

Available since 2.0

Broadcast infix operators

`vector operator+(vector x, real y)`

The result of adding y to every entry in the vector x

Available since 2.0

`vector operator+(real x, vector y)`

The result of adding x to every entry in the vector y

Available since 2.0

`row_vector operator+(row_vector x, real y)`

The result of adding y to every entry in the row vector x

Available since 2.0

`row_vector operator+(real x, row_vector y)`

The result of adding x to every entry in the row vector y

Available since 2.0

`matrix operator+(matrix x, real y)`

The result of adding y to every entry in the matrix x

Available since 2.0

`matrix operator+(real x, matrix y)`

The result of adding x to every entry in the matrix y

Available since 2.0

`vector operator-(vector x, real y)`

The result of subtracting y from every entry in the vector x

Available since 2.0

`vector operator-(real x, vector y)`

The result of adding x to every entry in the negation of the vector y

Available since 2.0

`row_vector operator-(row_vector x, real y)`

The result of subtracting y from every entry in the row vector x

Available since 2.0

`row_vector operator-(real x, row_vector y)`

The result of adding x to every entry in the negation of the row vector y

Available since 2.0

`matrix operator-(matrix x, real y)`

The result of subtracting y from every entry in the matrix x

Available since 2.0

`matrix operator-(real x, matrix y)`

The result of adding x to every entry in negation of the matrix y

Available since 2.0

`vector operator/(vector x, real y)`

The result of dividing each entry in the vector x by y

Available since 2.0

`row_vector operator/(row_vector x, real y)`

The result of dividing each entry in the row vector x by y

Available since 2.0

`matrix operator/(matrix x, real y)`

The result of dividing each entry in the matrix x by y

Available since 2.0

6.3. Transposition operator

Matrix transposition is represented using a postfix operator.

`matrix operator'(matrix x)`

The transpose of the matrix x, written as x'

Available since 2.0

`row_vector operator'(vector x)`

The transpose of the vector x, written as x'

Available since 2.0

`vector operator'(row_vector x)`

The transpose of the row vector x, written as x'

Available since 2.0

6.4. Elementwise functions

Elementwise functions apply a function to each element of a vector or matrix, returning a result of the same shape as the argument. There are many functions that are vectorized in addition to the ad hoc cases listed in this section; see section [function vectorization](#) for the general cases.

`vector operator.*(vector x, vector y)`

The elementwise product of y and x

Available since 2.0

`row_vector operator.*(row_vector x, row_vector y)`

The elementwise product of y and x

Available since 2.0

`matrix operator.*(matrix x, matrix y)`

The elementwise product of y and x

Available since 2.0

`vector operator./(vector x, vector y)`

The elementwise quotient of y and x

Available since 2.0

`vector operator./(vector x, real y)`

The elementwise quotient of y and x

Available since 2.4

`vector operator./(real x, vector y)`

The elementwise quotient of y and x

Available since 2.4

`row_vector operator./(row_vector x, row_vector y)`

The elementwise quotient of y and x

Available since 2.0

`row_vector operator./(row_vector x, real y)`

The elementwise quotient of y and x

Available since 2.4

`row_vector operator./(real x, row_vector y)`

The elementwise quotient of y and x

Available since 2.4

`matrix operator./(matrix x, matrix y)`

The elementwise quotient of y and x

Available since 2.0

`matrix operator./(matrix x, real y)`

The elementwise quotient of y and x

Available since 2.4

`matrix operator./(real x, matrix y)`

The elementwise quotient of y and x

Available since 2.4

`vector operator.^(vector x, vector y)`

The elementwise power of y and x

Available since 2.24

`vector operator.^(vector x, real y)`

The elementwise power of y and x

Available since 2.24

`vector operator.^(real x, vector y)`

The elementwise power of y and x

Available since 2.24

`row_vector operator.^(row_vector x, row_vector y)`

The elementwise power of y and x

Available since 2.24

`row_vector operator.^(row_vector x, real y)`

The elementwise power of y and x

Available since 2.24

`row_vector operator.^(real x, row_vector y)`

The elementwise power of y and x

Available since 2.24

`matrix operator.^(matrix x, matrix y)`

The elementwise power of y and x

Available since 2.24

`matrix operator.^(matrix x, real y)`

The elementwise power of y and x

Available since 2.24

`matrix operator.^(real x, matrix y)`

The elementwise power of y and x

Available since 2.24

6.5. Dot products and specialized products

`real dot_product(vector x, vector y)`

The dot product of x and y

Available since 2.0

`real dot_product(vector x, row_vector y)`

The dot product of x and y

Available since 2.0

`real dot_product(row_vector x, vector y)`

The dot product of x and y

Available since 2.0

`real dot_product(row_vector x, row_vector y)`

The dot product of x and y

Available since 2.0

`row_vector columns_dot_product(vector x, vector y)`

The dot product of the columns of x and y

Available since 2.0

`row_vector columns_dot_product(row_vector x, row_vector y)`

The dot product of the columns of x and y

Available since 2.0

`row_vector columns_dot_product(matrix x, matrix y)`

The dot product of the columns of x and y

Available since 2.0

`vector rows_dot_product(vector x, vector y)`

The dot product of the rows of x and y

Available since 2.0

`vector rows_dot_product(row_vector x, row_vector y)`

The dot product of the rows of x and y

Available since 2.0

`vector rows_dot_product(matrix x, matrix y)`

The dot product of the rows of x and y

Available since 2.0

real dot_self(vector *x*)

The dot product of the vector *x* with itself

Available since 2.0

real dot_self(row_vector *x*)

The dot product of the row vector *x* with itself

Available since 2.0

row_vector columns_dot_self(vector *x*)

The dot product of the columns of *x* with themselves

Available since 2.0

row_vector columns_dot_self(row_vector *x*)

The dot product of the columns of *x* with themselves

Available since 2.0

row_vector columns_dot_self(matrix *x*)

The dot product of the columns of *x* with themselves

Available since 2.0

vector rows_dot_self(vector *x*)

The dot product of the rows of *x* with themselves

Available since 2.0

vector rows_dot_self(row_vector *x*)

The dot product of the rows of *x* with themselves

Available since 2.0

vector rows_dot_self(matrix *x*)

The dot product of the rows of *x* with themselves

Available since 2.0

Specialized products

matrix tcrossprod(matrix *x*)

The product of *x* postmultiplied by its own transpose, similar to the `tcrossprod(x)` function in R. The result is a symmetric matrix xx^\top .

Available since 2.0

```
matrix crossprod(matrix x)
```

The product of x premultiplied by its own transpose, similar to the `crossprod(x)` function in R. The result is a symmetric matrix $x^\top x$.

Available since 2.0

The following functions all provide shorthand forms for common expressions, which are also much more efficient.

```
matrix quad_form(matrix A, matrix B)
```

The quadratic form, i.e., $B' * A * B$.

Available since 2.0

```
real quad_form(matrix A, vector B)
```

The quadratic form, i.e., $B' * A * B$.

Available since 2.0

```
matrix quad_form_diag(matrix m, vector v)
```

The quadratic form using the column vector v as a diagonal matrix, i.e., `diag_matrix(v) * m * diag_matrix(v)`.

Available since 2.3

```
matrix quad_form_diag(matrix m, row_vector rv)
```

The quadratic form using the row vector rv as a diagonal matrix, i.e., `diag_matrix(rv) * m * diag_matrix(rv)`.

Available since 2.3

```
matrix quad_form_sym(matrix A, matrix B)
```

Similarly to `quad_form`, gives $B' * A * B$, but additionally checks if A is symmetric and ensures that the result is also symmetric.

Available since 2.3

```
real quad_form_sym(matrix A, vector B)
```

Similarly to `quad_form`, gives $B' * A * B$, but additionally checks if A is symmetric and ensures that the result is also symmetric.

Available since 2.3

```
real trace_quad_form(matrix A, matrix B)
```

The trace of the quadratic form, i.e., `trace(B' * A * B)`.

Available since 2.0

real `trace_quad_form`(matrix A, vector B)

The trace of the quadratic form, i.e., $\text{trace}(B' * A * B)$.

Available since 2.0

real `trace_gen_quad_form`(matrix D, matrix A, matrix B)

The trace of a generalized quadratic form, i.e., $\text{trace}(D * B' * A * B)$.

Available since 2.0

matrix `multiply_lower_tri_self_transpose`(matrix x)

The product of the lower triangular portion of x (including the diagonal) times its own transpose; that is, if L is a matrix of the same dimensions as x with $L(m, n)$ equal to $x(m, n)$ for $n \leq m$ and $L(m, n)$ equal to 0 if $n > m$, the result is the symmetric matrix LL^\top . This is a specialization of `tcrossprod(x)` for lower-triangular matrices. The input matrix does not need to be square.

Available since 2.0

matrix `diag_pre_multiply`(vector v, matrix m)

Return the product of the diagonal matrix formed from the vector v and the matrix m , i.e., $\text{diag_matrix}(v) * m$.

Available since 2.0

matrix `diag_pre_multiply`(row_vector rv, matrix m)

Return the product of the diagonal matrix formed from the vector rv and the matrix m , i.e., $\text{diag_matrix}(rv) * m$.

Available since 2.0

matrix `diag_post_multiply`(matrix m, vector v)

Return the product of the matrix m and the diagonal matrix formed from the vector v , i.e., $m * \text{diag_matrix}(v)$.

Available since 2.0

matrix `diag_post_multiply`(matrix m, row_vector rv)

Return the product of the matrix m and the diagonal matrix formed from the the row vector rv , i.e., $m * \text{diag_matrix}(rv)$.

Available since 2.0

6.6. Reductions

Log sum of exponents

`real log_sum_exp(vector x)`

The natural logarithm of the sum of the exponentials of the elements in x

Available since 2.0

`real log_sum_exp(row_vector x)`

The natural logarithm of the sum of the exponentials of the elements in x

Available since 2.0

`real log_sum_exp(matrix x)`

The natural logarithm of the sum of the exponentials of the elements in x

Available since 2.0

Minimum and maximum

`real min(vector x)`

The minimum value in x , or $+\infty$ if x is empty

Available since 2.0

`real min(row_vector x)`

The minimum value in x , or $+\infty$ if x is empty

Available since 2.0

`real min(matrix x)`

The minimum value in x , or $+\infty$ if x is empty

Available since 2.0

`real max(vector x)`

The maximum value in x , or $-\infty$ if x is empty

Available since 2.0

`real max(row_vector x)`

The maximum value in x , or $-\infty$ if x is empty

Available since 2.0

`real max(matrix x)`

The maximum value in x , or $-\infty$ if x is empty

Available since 2.0

Sums and products

`real sum(vector x)`

The sum of the values in x , or 0 if x is empty

Available since 2.0

`real sum(row_vector x)`

The sum of the values in x , or 0 if x is empty

Available since 2.0

`real sum(matrix x)`

The sum of the values in x , or 0 if x is empty

Available since 2.0

`real prod(vector x)`

The product of the values in x , or 1 if x is empty

Available since 2.0

`real prod(row_vector x)`

The product of the values in x , or 1 if x is empty

Available since 2.0

`real prod(matrix x)`

The product of the values in x , or 1 if x is empty

Available since 2.0

Sample moments

Full definitions are provided for sample moments in section [array reductions](#).

`real mean(vector x)`

The sample mean of the values in x ; see section [array reductions](#) for details.

Available since 2.0

`real mean(row_vector x)`

The sample mean of the values in x ; see section [array reductions](#) for details.

Available since 2.0

`real mean(matrix x)`

The sample mean of the values in x ; see section [array reductions](#) for details.

Available since 2.0

```
real variance(vector x)
```

The sample variance of the values in x; see section [array reductions](#) for details.

Available since 2.0

```
real variance(row_vector x)
```

The sample variance of the values in x; see section [array reductions](#) for details.

Available since 2.0

```
real variance(matrix x)
```

The sample variance of the values in x; see section [array reductions](#) for details.

Available since 2.0

```
real sd(vector x)
```

The sample standard deviation of the values in x; see section [array reductions](#) for details.

Available since 2.0

```
real sd(row_vector x)
```

The sample standard deviation of the values in x; see section [array reductions](#) for details.

Available since 2.0

```
real sd(matrix x)
```

The sample standard deviation of the values in x; see section [array reductions](#) for details.

Available since 2.0

Quantile

Produces sample quantiles corresponding to the given probabilities. The smallest observation corresponds to a probability of 0 and the largest to a probability of 1.

Implements algorithm 7 from Hyndman, R. J. and Fan, Y., Sample quantiles in Statistical Packages (R's default quantile function).

```
real quantile(data vector x, data real p)
```

The p-th quantile of x

Available since 2.27

```
array[] real quantile(data vector x, data array[] real p)
```

An array containing the quantiles of x given by the array of probabilities p

Available since 2.27

`real quantile(data row_vector x, data real p)`

The p-th quantile of x

Available since 2.27

`array[] real quantile(data row_vector x, data array[] real p)`

An array containing the quantiles of x given by the array of probabilities p

Available since 2.27

6.7. Broadcast functions

The following broadcast functions allow vectors, row vectors and matrices to be created by copying a single element into all of their cells. Matrices may also be created by stacking copies of row vectors vertically or stacking copies of column vectors horizontally.

`vector rep_vector(real x, int m)`

Return the size m (column) vector consisting of copies of x.

Available since 2.0

`row_vector rep_row_vector(real x, int n)`

Return the size n row vector consisting of copies of x.

Available since 2.0

`matrix rep_matrix(real x, int m, int n)`

Return the m by n matrix consisting of copies of x.

Available since 2.0

`matrix rep_matrix(vector v, int n)`

Return the m by n matrix consisting of n copies of the (column) vector v of size m.

Available since 2.0

`matrix rep_matrix(row_vector rv, int m)`

Return the m by n matrix consisting of m copies of the row vector rv of size n.

Available since 2.0

Unlike the situation with array broadcasting (see section [array broadcasting](#)), where there is a distinction between integer and real arguments, the following two statements produce the same result for vector broadcasting; row vector and matrix broadcasting behave similarly.

```
vector[3] x;  
x = rep_vector(1, 3);  
x = rep_vector(1.0, 3);
```

There are no integer vector or matrix types, so integer values are automatically promoted.

Symmetrization

```
matrix symmetrize_from_lower_tri(matrix A)
```

Construct a symmetric matrix from the lower triangle of A.

Available since 2.26

6.8. Diagonal matrix functions

```
matrix add_diag(matrix m, row_vector d)
```

Add row_vector d to the diagonal of matrix m.

Available since 2.21

```
matrix add_diag(matrix m, vector d)
```

Add vector d to the diagonal of matrix m.

Available since 2.21

```
matrix add_diag(matrix m, real d)
```

Add scalar d to every diagonal element of matrix m.

Available since 2.21

```
vector diagonal(matrix x)
```

The diagonal of the matrix x

Available since 2.0

```
matrix diag_matrix(vector x)
```

The diagonal matrix with diagonal x

Available since 2.0

Although the `diag_matrix` function is available, it is unlikely to ever show up in an efficient Stan program. For example, rather than converting a diagonal to a full matrix for use as a covariance matrix,

```
y ~ multi_normal(mu, diag_matrix(square(sigma)));
```

it is much more efficient to just use a univariate normal, which produces the same density,

```
y ~ normal(mu, sigma);
```

Rather than writing `m * diag_matrix(v)` where `m` is a matrix and `v` is a vector, it is much more efficient to write `diag_post_multiply(m, v)` (and similarly for pre-multiplication). By the same token, it is better to use `quad_form_diag(m, v)` rather than `quad_form(m, diag_matrix(v))`.

`matrix identity_matrix(int k)`

Create an identity matrix of size $k \times k$

Available since 2.26

6.9. Container construction functions

`array[] real linspaced_array(int n, data real lower, data real upper)`

Create a real array of length `n` of equidistantly-spaced elements between `lower` and `upper`

Available since 2.24

`array[] int linspaced_int_array(int n, int lower, int upper)`

Create a regularly spaced, increasing integer array of length `n` between `lower` and `upper`, inclusively. If $(upper - lower) / (n - 1)$ is less than one, repeat each output $(n - 1) / (upper - lower)$ times. If neither $(upper - lower) / (n - 1)$ or $(n - 1) / (upper - lower)$ are integers, `upper` is reduced until one of these is true.

Available since 2.26

`vector linspaced_vector(int n, data real lower, data real upper)`

Create an n -dimensional vector of equidistantly-spaced elements between `lower` and `upper`

Available since 2.24

`row_vector linspaced_row_vector(int n, data real lower, data real upper)`

Create an n -dimensional row-vector of equidistantly-spaced elements between `lower` and `upper`

Available since 2.24

`array[] int one_hot_int_array(int n, int k)`

Create a one-hot encoded int array of length n with `array[k] = 1`

Available since 2.26

`array[] real one_hot_array(int n, int k)`

Create a one-hot encoded real array of length n with `array[k] = 1`

Available since 2.24

`vector one_hot_vector(int n, int k)`

Create an n-dimensional one-hot encoded vector with `vector[k] = 1`

Available since 2.24

`row_vector one_hot_row_vector(int n, int k)`

Create an n-dimensional one-hot encoded row-vector with `row_vector[k] = 1`

Available since 2.24

`array[] int ones_int_array(int n)`

Create an int array of length n of all ones

Available since 2.26

`array[] real ones_array(int n)`

Create a real array of length n of all ones

Available since 2.26

`vector ones_vector(int n)`

Create an n-dimensional vector of all ones

Available since 2.26

`row_vector ones_row_vector(int n)`

Create an n-dimensional row-vector of all ones

Available since 2.26

`array[] int zeros_int_array(int n)`

Create an int array of length n of all zeros

Available since 2.26

`array[] real zeros_array(int n)`

Create a real array of length n of all zeros

Available since 2.24

`vector zeros_vector(int n)`

Create an n -dimensional vector of all zeros

Available since 2.24

`row_vector zeros_row_vector(int n)`

Create an n -dimensional row-vector of all zeros

Available since 2.24

`vector uniform_simplex(int n)`

Create an n -dimensional simplex with elements `vector[i] = 1 / n` for all $i \in 1, \dots, n$

Available since 2.24

6.10. Slicing and blocking functions

Stan provides several functions for generating slices or blocks or diagonal entries for matrices.

Columns and rows

`vector col(matrix x, int n)`

The n -th column of matrix x

Available since 2.0

`row_vector row(matrix x, int m)`

The m -th row of matrix x

Available since 2.0

The `row` function is special in that it may be used as an lvalue in an assignment statement (i.e., something to which a value may be assigned). The `row` function is also special in that the indexing notation `x[m]` is just an alternative way of writing `row(x, m)`. The `col` function may **not** be used as an lvalue, nor is there an indexing based shorthand for it.

Block operations

Matrix slicing operations

Block operations may be used to extract a sub-block of a matrix.

`matrix block(matrix x, int i, int j, int n_rows, int n_cols)`

Return the submatrix of x that starts at row i and column j and extends n_{rows}

rows and `n_cols` columns.

Available since 2.0

The sub-row and sub-column operations may be used to extract a slice of row or column from a matrix

`vector sub_col(matrix x, int i, int j, int n_rows)`

Return the sub-column of `x` that starts at row `i` and column `j` and extends `n_rows` rows and 1 column.

Available since 2.0

`row_vector sub_row(matrix x, int i, int j, int n_cols)`

Return the sub-row of `x` that starts at row `i` and column `j` and extends 1 row and `n_cols` columns.

Available since 2.0

Vector and array slicing operations

The `head` operation extracts the first `n` elements of a vector and the `tail` operation the last. The `segment` operation extracts an arbitrary subvector.

`vector head(vector v, int n)`

Return the vector consisting of the first `n` elements of `v`.

Available since 2.0

`row_vector head(row_vector rv, int n)`

Return the row vector consisting of the first `n` elements of `rv`.

Available since 2.0

`array[] T head(array[] T sv, int n)`

Return the array consisting of the first `n` elements of `sv`; applies to up to three-dimensional arrays containing any type of elements `T`.

Available since 2.0

`vector tail(vector v, int n)`

Return the vector consisting of the last `n` elements of `v`.

Available since 2.0

`row_vector tail(row_vector rv, int n)`

Return the row vector consisting of the last `n` elements of `rv`.

Available since 2.0

`array[] T tail(array[] T sv, int n)`

Return the array consisting of the last n elements of sv ; applies to up to three-dimensional arrays containing any type of elements T .

Available since 2.0

`vector segment(vector v, int i, int n)`

Return the vector consisting of the n elements of v starting at i ; i.e., elements i through $i + n - 1$.

Available since 2.0

`row_vector segment(row_vector rv, int i, int n)`

Return the row vector consisting of the n elements of rv starting at i ; i.e., elements i through $i + n - 1$.

Available since 2.10

`array[] T segment(array[] T sv, int i, int n)`

Return the array consisting of the n elements of sv starting at i ; i.e., elements i through $i + n - 1$. Applies to up to three-dimensional arrays containing any type of elements T .

Available since 2.0

6.11. Matrix and vector concatenation

Stan's matrix and vector concatenation operations `append_col` and `append_row` are like the operations `cbind` and `rbind` in R.

Horizontal concatenation

`matrix append_col(matrix x, matrix y)`

Combine matrices x and y by column. The matrices must have the same number of rows.

Available since 2.5

`matrix append_col(matrix x, vector y)`

Combine matrix x and vector y by column. The matrix and the vector must have the same number of rows.

Available since 2.5

`matrix append_col(vector x, matrix y)`

Combine vector x and matrix y by column. The vector and the matrix must have the same number of rows.

Available since 2.5

matrix `append_col`(vector x, vector y)

Combine vectors x and y by column. The vectors must have the same number of rows.

Available since 2.5

row_vector `append_col`(row_vector x, row_vector y)

Combine row vectors x and y of any size into another row vector by appending y to the end of x.

Available since 2.5

row_vector `append_col`(real x, row_vector y)

Append x to the front of y, returning another row vector.

Available since 2.12

row_vector `append_col`(row_vector x, real y)

Append y to the end of x, returning another row vector.

Available since 2.12

Vertical concatenation

matrix `append_row`(matrix x, matrix y)

Combine matrices x and y by row. The matrices must have the same number of columns.

Available since 2.5

matrix `append_row`(matrix x, row_vector y)

Combine matrix x and row vector y by row. The matrix and the row vector must have the same number of columns.

Available since 2.5

matrix `append_row`(row_vector x, matrix y)

Combine row vector x and matrix y by row. The row vector and the matrix must have the same number of columns.

Available since 2.5

matrix `append_row`(row_vector x, row_vector y)

Combine row vectors x and y by row. The row vectors must have the same number of columns.

Available since 2.5

vector `append_row`(vector x , vector y)

Concatenate vectors x and y of any size into another vector.

Available since 2.5

vector `append_row`(real x , vector y)

Append x to the top of y , returning another vector.

Available since 2.12

vector `append_row`(vector x , real y)

Append y to the bottom of x , returning another vector.

Available since 2.12

6.12. Special matrix functions

Softmax

The softmax function maps¹ $y \in \mathbb{R}^K$ to the K -simplex by

$$\text{softmax}(y) = \frac{\exp(y)}{\sum_{k=1}^K \exp(y_k)},$$

where $\exp(y)$ is the componentwise exponentiation of y . Softmax is usually calculated on the log scale,

$$\begin{aligned} \log \text{softmax}(y) &= y - \log \sum_{k=1}^K \exp(y_k) \\ &= y - \log_sum_exp(y). \end{aligned}$$

where the vector y minus the scalar $\log_sum_exp(y)$ subtracts the scalar from each component of y .

Stan provides the following functions for softmax and its log.

vector `softmax`(vector x)

The softmax of x

Available since 2.0

¹The softmax function is so called because in the limit as $y_n \rightarrow \infty$ with y_m for $m \neq n$ held constant, the result tends toward the “one-hot” vector θ with $\theta_n = 1$ and $\theta_m = 0$ for $m \neq n$, thus providing a “soft” version of the maximum function.

`vector log_softmax(vector x)`

The natural logarithm of the softmax of x

Available since 2.0

Cumulative sums

The cumulative sum of a sequence x_1, \dots, x_N is the sequence y_1, \dots, y_N , where

$$y_n = \sum_{m=1}^n x_m.$$

`array[] int cumulative_sum(array[] int x)`

The cumulative sum of x

Available since 2.30

`array[] real cumulative_sum(array[] real x)`

The cumulative sum of x

Available since 2.0

`vector cumulative_sum(vector v)`

The cumulative sum of v

Available since 2.0

`row_vector cumulative_sum(row_vector rv)`

The cumulative sum of rv

Available since 2.0

6.13. Gaussian Process Covariance Functions

The Gaussian process covariance functions compute the covariance between observations in an input data set or the cross-covariance between two input data sets.

For one dimensional GPs, the input data sets are arrays of scalars. The covariance matrix is given by $K_{ij} = k(x_i, x_j)$ (where x_i is the i^{th} element of the array x) and the cross-covariance is given by $K_{ij} = k(x_i, y_j)$.

For multi-dimensional GPs, the input data sets are arrays of vectors. The covariance matrix is given by $K_{ij} = k(\mathbf{x}_i, \mathbf{x}_j)$ (where \mathbf{x}_i is the i^{th} vector in the array x) and the cross-covariance is given by $K_{ij} = k(\mathbf{x}_i, \mathbf{y}_j)$.

Exponentiated quadratic kernel

With magnitude σ and length scale l , the exponentiated quadratic kernel is:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \exp\left(-\frac{|\mathbf{x}_i - \mathbf{x}_j|^2}{2l^2}\right)$$

```
matrix          gp_exp_quad_cov(array[] real x, real sigma, real
length_scale)
```

Gaussian process covariance with exponentiated quadratic kernel in one dimension.

Available since 2.20

```
matrix          gp_exp_quad_cov(array[] real x1, array[] real x2, real
sigma, real length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with exponentiated quadratic kernel in one dimension.

Available since 2.20

```
matrix gp_exp_quad_cov(vectors x, real sigma, real length_scale)
```

Gaussian process covariance with exponentiated quadratic kernel in multiple dimensions.

Available since 2.20

```
matrix          gp_exp_quad_cov(vectors x, real sigma, array[] real
length_scale)
```

Gaussian process covariance with exponentiated quadratic kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

```
matrix          gp_exp_quad_cov(vectors x1, vectors x2, real sigma, real
length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with exponentiated quadratic kernel in multiple dimensions.

Available since 2.20

```
matrix gp_exp_quad_cov(vectors x1, vectors x2, real sigma, array[]  
real length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with exponentiated quadratic kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

Dot product kernel

With bias σ_0 the dot product kernel is:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sigma_0^2 + \mathbf{x}_i^T \mathbf{x}_j$$

```
matrix gp_dot_prod_cov(array[] real x, real sigma)
```

Gaussian process covariance with dot product kernel in one dimension.

Available since 2.20

```
matrix      gp_dot_prod_cov(array[] real x1, array[] real x2, real  
sigma)
```

Gaussian process cross-covariance of x_1 and x_2 with dot product kernel in one dimension.

Available since 2.20

```
matrix gp_dot_prod_cov(vectors x, real sigma)
```

Gaussian process covariance with dot product kernel in multiple dimensions.

Available since 2.20

```
matrix gp_dot_prod_cov(vectors x1, vectors x2, real sigma)
```

Gaussian process cross-covariance of x_1 and x_2 with dot product kernel in multiple dimensions.

Available since 2.20

Exponential kernel

With magnitude σ and length scale l , the exponential kernel is:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \exp\left(-\frac{|\mathbf{x}_i - \mathbf{x}_j|}{l}\right)$$

```
matrix      gp_exponential_cov(array[] real x, real sigma, real
length_scale)
```

Gaussian process covariance with exponential kernel in one dimension.

Available since 2.20

```
matrix      gp_exponential_cov(array[] real x1, array[] real x2, real
sigma, real length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with exponential kernel in one dimension.

Available since 2.20

```
matrix gp_exponential_cov(vectors x, real sigma, real length_scale)
```

Gaussian process covariance with exponential kernel in multiple dimensions.

Available since 2.20

```
matrix      gp_exponential_cov(vectors x, real sigma, array[] real
length_scale)
```

Gaussian process covariance with exponential kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

```
matrix gp_exponential_cov(vectors x1, vectors x2, real sigma, real
length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with exponential kernel in multiple dimensions.

Available since 2.20

```
matrix gp_exponential_cov(vectors x1, vectors x2, real sigma, array[] real
length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with exponential kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

Matern 3/2 kernel

With magnitude σ and length scale l , the Matern 3/2 kernel is:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \left(1 + \frac{\sqrt{3}|\mathbf{x}_i - \mathbf{x}_j|}{l} \right) \exp \left(-\frac{\sqrt{3}|\mathbf{x}_i - \mathbf{x}_j|}{l} \right)$$

```
matrix gp_matern32_cov(array[] real x, real sigma, real
length_scale)
```

Gaussian process covariance with Matern 3/2 kernel in one dimension.

Available since 2.20

```
matrix gp_matern32_cov(array[] real x1, array[] real x2, real
sigma, real length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with Matern 3/2 kernel in one dimension.

Available since 2.20

```
matrix gp_matern32_cov(vectors x, real sigma, real length_scale)
```

Gaussian process covariance with Matern 3/2 kernel in multiple dimensions.

Available since 2.20

```
matrix      gp_matern32_cov(vectors x, real sigma, array[] real
length_scale)
```

Gaussian process covariance with Matern 3/2 kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

```
matrix      gp_matern32_cov(vectors x1, vectors x2, real sigma, real
length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with Matern 3/2 kernel in multiple dimensions.

Available since 2.20

```
matrix  gp_matern32_cov(vectors x1, vectors x2, real sigma, array[] real
length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with Matern 3/2 kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

Matern 5/2 kernel

With magnitude σ and length scale l , the Matern 5/2 kernel is:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \left(1 + \frac{\sqrt{5}|\mathbf{x}_i - \mathbf{x}_j|}{l} + \frac{5|\mathbf{x}_i - \mathbf{x}_j|^2}{3l^2} \right) \exp \left(-\frac{\sqrt{5}|\mathbf{x}_i - \mathbf{x}_j|}{l} \right)$$

```
matrix      gp_matern52_cov(array[] real x, real sigma, real
length_scale)
```

Gaussian process covariance with Matern 5/2 kernel in one dimension.

Available since 2.20

```
matrix      gp_matern52_cov(array[] real x1, array[] real x2, real
sigma, real length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with Matern 5/2 kernel in one dimension.

Available since 2.20

```
matrix gp_matern52_cov(vectors x, real sigma, real length_scale)
```

Gaussian process covariance with Matern 5/2 kernel in multiple dimensions.

Available since 2.20

```
matrix          gp_matern52_cov(vectors x, real sigma, array[] real
length_scale)
```

Gaussian process covariance with Matern 5/2 kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

```
matrix      gp_matern52_cov(vectors x1, vectors x2, real sigma, real
length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with Matern 5/2 kernel in multiple dimensions.

Available since 2.20

```
matrix  gp_matern52_cov(vectors x1, vectors x2, real sigma, array[]
real length_scale)
```

Gaussian process cross-covariance of x_1 and x_2 with Matern 5/2 kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

Periodic kernel

With magnitude σ , length scale l , and period p , the periodic kernel is:

$$k(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 \exp \left(-\frac{2 \sin^2 \left(\pi \frac{|\mathbf{x}_i - \mathbf{x}_j|}{p} \right)}{l^2} \right)$$

```
matrix      gp_periodic_cov(array[] real x, real sigma, real
length_scale, real period)
```

Gaussian process covariance with periodic kernel in one dimension.

Available since 2.20

```
matrix      gp_periodic_cov(array[] real x1, array[] real x2, real
sigma, real length_scale, real period)
```

Gaussian process cross-covariance of x_1 and x_2 with periodic kernel in one dimension.

Available since 2.20

```
matrix      gp_periodic_cov(vectors x, real sigma, real length_scale,
real period)
```

Gaussian process covariance with periodic kernel in multiple dimensions.

Available since 2.20

```
matrix      gp_periodic_cov(vectors x1, vectors x2, real sigma, real
length_scale, real period)
```

Gaussian process cross-covariance of x_1 and x_2 with periodic kernel in multiple dimensions with a length scale for each dimension.

Available since 2.20

6.14. Linear algebra functions and solvers

Matrix division operators and functions

In general, it is much more efficient and also more arithmetically stable to use matrix division than to multiply by an inverse. There are specialized forms for lower triangular matrices and for symmetric, positive-definite matrices.

Matrix division operators

`row_vector operator/(row_vector b, matrix A)`

The right division of b by A; equivalently `b * inverse(A)`

Available since 2.0

`matrix operator/(matrix B, matrix A)`

The right division of B by A; equivalently `B * inverse(A)`

Available since 2.5

`vector operator\ (matrix A, vector b)`

The left division of A by b; equivalently `inverse(A) * b`

Available since 2.18

`matrix operator\ (matrix A, matrix B)`

The left division of A by B; equivalently `inverse(A) * B`

Available since 2.18

Lower-triangular matrix division functions

There are four division functions which use lower triangular views of a matrix. The lower triangular view of a matrix `tri(A)` is used in the definitions and defined by

$$\text{tri}(A)[m, n] = \begin{cases} A[m, n] & \text{if } m \geq n, \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

When a lower triangular view of a matrix is used, the elements above the diagonal are ignored.

`vector mdivide_left_tri_low(matrix A, vector b)`

The left division of b by a lower-triangular view of A; algebraically equivalent to the less efficient and stable form `inverse(tri(A)) * b`, where `tri(A)` is the lower-triangular portion of A with the above-diagonal entries set to zero.

Available since 2.12

`matrix mdivide_left_tri_low(matrix A, matrix B)`

The left division of B by a triangular view of A; algebraically equivalent to the less efficient and stable form `inverse(tri(A)) * B`, where `tri(A)` is the lower-triangular portion of A with the above-diagonal entries set to zero.

Available since 2.5

`row_vector mdivide_right_tri_low(row_vector b, matrix A)`

The right division of b by a triangular view of A ; algebraically equivalent to the less efficient and stable form $b * \text{inverse}(\text{tri}(A))$, where $\text{tri}(A)$ is the lower-triangular portion of A with the above-diagonal entries set to zero.

Available since 2.12

`matrix mdivide_right_tri_low(matrix B, matrix A)`

The right division of B by a triangular view of A ; algebraically equivalent to the less efficient and stable form $B * \text{inverse}(\text{tri}(A))$, where $\text{tri}(A)$ is the lower-triangular portion of A with the above-diagonal entries set to zero.

Available since 2.5

Symmetric positive-definite matrix division functions

There are four division functions which are specialized for efficiency and stability for symmetric positive-definite matrix dividends. If the matrix dividend argument is not symmetric and positive definite, these will reject and print warnings.

`matrix mdivide_left_spd(matrix A, vector b)`

The left division of b by the symmetric, positive-definite matrix A ; algebraically equivalent to the less efficient and stable form $\text{inverse}(A) * b$.

Available since 2.12

`vector mdivide_left_spd(matrix A, matrix B)`

The left division of B by the symmetric, positive-definite matrix A ; algebraically equivalent to the less efficient and stable form $\text{inverse}(A) * B$.

Available since 2.12

`row_vector mdivide_right_spd(row_vector b, matrix A)`

The right division of b by the symmetric, positive-definite matrix A ; algebraically equivalent to the less efficient and stable form $b * \text{inverse}(A)$.

Available since 2.12

`matrix mdivide_right_spd(matrix B, matrix A)`

The right division of B by the symmetric, positive-definite matrix A ; algebraically equivalent to the less efficient and stable form $B * \text{inverse}(A)$.

Available since 2.12

Matrix exponential

The exponential of the matrix A is formally defined by the convergent power series:

$$e^A = \sum_{n=0}^{\infty} \frac{A^n}{n!}$$

matrix `matrix_exp`(matrix A)

The matrix exponential of A

Available since 2.13

matrix `matrix_exp_multiply`(matrix A, matrix B)

The multiplication of matrix exponential of A and matrix B; algebraically equivalent to the less efficient form `matrix_exp(A) * B`.

Available since 2.18

matrix `scale_matrix_exp_multiply`(real t, matrix A, matrix B)

The multiplication of matrix exponential of tA and matrix B; algebraically equivalent to the less efficient form `matrix_exp(t * A) * B`.

Available since 2.18

Matrix power

Returns the n th power of the specific matrix:

$$M^n = M_1 * \dots * M_n$$

matrix `matrix_power`(matrix A, int B)

Matrix A raised to the power B.

Available since 2.24

Linear algebra functions

Trace

real `trace`(matrix A)

The trace of A, or 0 if A is empty; A is not required to be diagonal

Available since 2.0

Determinants

real `determinant`(matrix A)

The determinant of A

Available since 2.0

real log_determinant(matrix A)

The log of the absolute value of the determinant of A

Available since 2.0

real log_determinant_spd(matrix A)

The log of the absolute value of the determinant of the symmetric, positive-definite matrix A.

Available since 2.30

Inverses

It is almost never a good idea to use matrix inverses directly because they are both inefficient and arithmetically unstable compared to the alternatives. Rather than inverting a matrix m and post-multiplying by a vector or matrix a , as in `inverse(m) * a`, it is better to code this using matrix division, as in $m \setminus a$. The pre-multiplication case is similar, with $b * inverse(m)$ being more efficiently coded as b / m . There are also useful special cases for triangular and symmetric, positive-definite matrices that use more efficient solvers.

Warning: The function `inv(m)` is the elementwise inverse function, which returns $1 / m[i, j]$ for each element.

matrix inverse(matrix A)

Compute the inverse of A

Available since 2.0

matrix inverse_spd(matrix A)

Compute the inverse of A where A is symmetric, positive definite. This version is faster and more arithmetically stable when the input is symmetric and positive definite.

Available since 2.0

matrix chol2inv(matrix L)

Compute the inverse of the matrix whose cholesky factorization is L. That is, for $A = LL^T$, return A^{-1} .

Available since 2.26

Generalized Inverse

The generalized inverse M^+ of a matrix M is a matrix that satisfies $MM^+M = M$. For an invertible, square matrix M , M^+ is equivalent to M^{-1} . The dimensions of

M^+ are equivalent to the dimensions of M^T . The generalized inverse exists for any matrix, so the M may be singular or less than full rank.

Even though the generalized inverse exists for any arbitrary matrix, the derivatives of this function only exist on matrices of locally constant rank (Golub and Pereyra 1973), meaning, the derivatives do not exist if small perturbations make the matrix change rank. For example, considered the rank of the matrix A as a function of ϵ :

$$A = \begin{pmatrix} 1 + \epsilon & 2 & 1 \\ 2 & 4 & 2 \end{pmatrix}$$

When $\epsilon = 0$, A is rank 1 because the second row is twice the first (and so there is only one linearly independent row). If $\epsilon \neq 0$, the rows are no longer linearly dependent, and the matrix is rank 2. This matrix does not have locally constant rank at $\epsilon = 0$, and so the derivatives do not exist at zero. Because HMC depends on the derivatives existing, this lack of differentiability creates undefined behavior.

`matrix generalized_inverse(matrix A)`

The generalized inverse of A

Available since 2.26

Eigendecomposition

`complex_vector eigenvalues(matrix A)`

The complex-valued vector of eigenvalues of the matrix A . The eigenvalues are repeated according to their algebraic multiplicity, so there are as many eigenvalues as rows in the matrix. The eigenvalues are not sorted in any particular order.

Available since 2.30

`complex_matrix eigenvectors(matrix A)`

The matrix with the complex-valued (column) eigenvectors of the matrix A in the same order as returned by the function `eigenvalues`

Available since 2.30

`tuple(complex_matrix, complex_vector) eigendecompose(matrix A)`

Return the matrix of (column) eigenvectors and vector of eigenvalues of the matrix A . This function is equivalent to `(eigenvectors(A), eigenvalues(A))` but with a lower computational cost due to the shared work between the two results.

Available since 2.33

`vector eigenvalues_sym(matrix A)`

The vector of eigenvalues of a symmetric matrix A in ascending order

Available since 2.0

`matrix eigenvectors_sym(matrix A)`

The matrix with the (column) eigenvectors of symmetric matrix A in the same order as returned by the function `eigenvalues_sym`

Available since 2.0

`tuple(matrix, vector) eigendecompose_sym(matrix A)`

Return the matrix of (column) eigenvectors and vector of eigenvalues of the symmetric matrix A. This function is equivalent to `(eigenvectors_sym(A), eigenvalues_sym(A))` but with a lower computational cost due to the shared work between the two results.

Available since 2.33

Because multiplying an eigenvector by -1 results in an eigenvector, eigenvectors returned by a decomposition are only identified up to a sign change. In order to compare the eigenvectors produced by Stan's eigendecomposition to others, signs may need to be normalized in some way, such as by fixing the sign of a component, or doing comparisons allowing a multiplication by -1 .

The condition number of a symmetric matrix is defined to be the ratio of the largest eigenvalue to the smallest eigenvalue. Large condition numbers lead to difficulty in numerical algorithms such as computing inverses, and thus known as "ill conditioned." The ratio can even be infinite in the case of singular matrices (i.e., those with eigenvalues of 0).

QR decomposition

`matrix qr_thin_Q(matrix A)`

The orthogonal matrix in the thin QR decomposition of A, which implies that the resulting matrix has the same dimensions as A

Available since 2.18

`matrix qr_thin_R(matrix A)`

The upper triangular matrix in the thin QR decomposition of A, which implies that the resulting matrix is square with the same number of columns as A

Available since 2.18

`tuple(matrix, matrix) qr_thin(matrix A)`

Returns both portions of the QR decomposition of A. The first element (“Q”) is the orthonormal matrix in the thin QR decomposition and the second element (“R”) is upper triangular. This function is equivalent to `(qr_thin_Q(A), qr_thin_R(A))` but with a lower computational cost due to the shared work between the two results.

Available since 2.33

`matrix qr_Q(matrix A)`

The orthogonal matrix in the fat QR decomposition of A, which implies that the resulting matrix is square with the same number of rows as A

Available since 2.3

`matrix qr_R(matrix A)`

The upper trapezoidal matrix in the fat QR decomposition of A, which implies that the resulting matrix will be rectangular with the same dimensions as A

Available since 2.3

`tuple(matrix, matrix) qr(matrix A)`

Returns both portions of the QR decomposition of A. The first element (“Q”) is the orthonormal matrix in the thin QR decomposition and the second element (“R”) is upper triangular. This function is equivalent to `(qr_Q(A), qr_R(A))` but with a lower computational cost due to the shared work between the two results.

Available since 2.33

The thin QR decomposition is always preferable because it will consume much less memory when the input matrix is large than will the fat QR decomposition. Both versions of the decomposition represent the input matrix as

$$A = Q R.$$

Multiplying a column of an orthogonal matrix by -1 still results in an orthogonal matrix, and you can multiply the corresponding row of the upper trapezoidal matrix by -1 without changing the product. Thus, Stan adopts the normalization that the diagonal elements of the upper trapezoidal matrix are strictly positive and the columns of the orthogonal matrix are reflected if necessary. Also, these QR decomposition algorithms do not utilize pivoting and thus may be numerically unstable on input matrices that have less than full rank.

Cholesky decomposition

Every symmetric, positive-definite matrix (such as a correlation or covariance matrix) has a Cholesky decomposition. If Σ is a symmetric, positive-definite matrix,

its Cholesky decomposition is the lower-triangular vector L such that

$$\Sigma = L L^\top.$$

matrix `cholesky_decompose`(matrix A)

The lower-triangular Cholesky factor of the symmetric positive-definite matrix A

Available since 2.0

Singular value decomposition

The matrix A can be decomposed into a diagonal matrix of singular values, D, and matrices of its left and right singular vectors, U and V,

$$A = UDV^T.$$

The matrices of singular vectors here are thin. That is for an N by P input A, $M = \min(N, P)$, U is size N by M and V is size P by M .

vector `singular_values`(matrix A)

The singular values of A in descending order

Available since 2.0

matrix `svd_U`(matrix A)

The left-singular vectors of A

Available since 2.26

matrix `svd_V`(matrix A)

The right-singular vectors of A

Available since 2.26

tuple(matrix, vector, matrix) `svd`(matrix A)

Returns a tuple containing the left-singular vectors of A, the singular values of A in descending order, and the right-singular values of A. This function is equivalent to `(svd_U(A), singular_values(A), svd_V(A))` but with a lower computational cost due to the shared work between the different components.

Available since 2.33

6.15. Sort functions

See the [sorting functions section](#) for examples of how the functions work.

vector `sort_asc`(vector v)

Sort the elements of v in ascending order

Available since 2.0

`row_vector sort_asc(row_vector v)`

Sort the elements of v in ascending order

Available since 2.0

`vector sort_desc(vector v)`

Sort the elements of v in descending order

Available since 2.0

`row_vector sort_desc(row_vector v)`

Sort the elements of v in descending order

Available since 2.0

`array[] int sort_indices_asc(vector v)`

Return an array of indices between 1 and the size of v, sorted to index v in ascending order.

Available since 2.3

`array[] int sort_indices_asc(row_vector v)`

Return an array of indices between 1 and the size of v, sorted to index v in ascending order.

Available since 2.3

`array[] int sort_indices_desc(vector v)`

Return an array of indices between 1 and the size of v, sorted to index v in descending order.

Available since 2.3

`array[] int sort_indices_desc(row_vector v)`

Return an array of indices between 1 and the size of v, sorted to index v in descending order.

Available since 2.3

`int rank(vector v, int s)`

Number of components of v less than v[s]

Available since 2.0

`int rank(row_vector v, int s)`

Number of components of v less than v[s]

Available since 2.0

6.16. Reverse functions

`vector reverse(vector v)`

Return a new vector containing the elements of the argument in reverse order.

Available since 2.23

`row_vector reverse(row_vector v)`

Return a new row vector containing the elements of the argument in reverse order.

Available since 2.23

7. Complex Matrix Operations

7.1. Complex promotion

This chapter provides the details of functions that operate over complex matrices, vectors, and row vectors. These mirror the operations over real `complex_matrix` types and are defined in the usual way for complex numbers.

Promotion of complex arguments

If an expression `e` can be assigned to a variable of type `T`, then it can be used as an argument to a function that is specified to take arguments of type `T`. For instance, `sqrt(real)` is specified to take a `real` argument, but an integer expression such as `2 + 2` of type `int` can be passed to `sqrt`, so that `sqrt(2 + 2)` is well defined. This works by promoting the integer expression `2 + 2` to be of `real` type.

The rules for promotion in Stan are simple:

- `int` may be promoted to `real`,
- `real` may be promoted to `complex`,
- `vector` can be promoted to `complex_vector`,
- `row_vector` can be promoted to `complex_row_vector`,
- `matrix` can be promoted to `complex_matrix`,
- if `T` can be promoted to `U` and `U` can be promoted to `V`, then `T` can be promoted to `V` (transitive), and
- if `T` can be promoted to `U`, then `T[]` can be promoted to `U[]` (covariant).

Signature selection

When a function is called, the definition requiring the fewest number of promotions is used. For example, when calling `vector + vector`, the real-valued signature is used. When calling any of `complex_vector + vector`, `vector + complex_vector`, or `complex_vector + complex_vector`, the complex signature is used. If more than one signature matches with the minimal number of promotions, the call is ambiguous, and an error will be raised by the compiler. Promotion ambiguity leading to ill-defined calls should never happen with Stan built-in functions.

Signatures for complex functions

Complex function signatures will only list the fully complex type. For example, with complex vector addition, we will list a single signature, `complex_operator+(complex_vector, complex_vector)`. Through promotion, `operator+` may

be called with one complex vector and one real vector as well, but the documentation elides the implied signatures `operator+(complex_vector, vector)` and `operator+(vector, complex_vector)`.

Generic functions work for complex containers

Generic functions work for arrays containing complex, complex matrix, complex vector, or complex row vector types. This includes the functions `append_array`, `dims`, `head`, `num_elements`, `rep_array`, `reverse`, `segment`, `size`, and `tail`.

7.2. Integer-valued complex matrix size functions

`int num_elements(complex_vector x)`

The total number of elements in the vector `x` (same as function `rows`)

Available since 2.30

`int num_elements(complex_row_vector x)`

The total number of elements in the vector `x` (same as function `cols`)

Available since 2.30

`int num_elements(complex_matrix x)`

The total number of elements in the matrix `x`. For example, if `x` is a 5×3 matrix, then `num_elements(x)` is 15

Available since 2.30

`int rows(complex_vector x)`

The number of rows in the vector `x`

Available since 2.30

`int rows(complex_row_vector x)`

The number of rows in the row vector `x`, namely 1

Available since 2.30

`int rows(complex_matrix x)`

The number of rows in the matrix `x`

Available since 2.30

`int cols(complex_vector x)`

The number of columns in the vector `x`, namely 1

Available since 2.30

```
int cols(complex_row_vector x)
```

The number of columns in the row vector x

Available since 2.30

```
int cols(complex_matrix x)
```

The number of columns in the matrix x

Available since 2.30

```
int size(complex_vector x)
```

The size of x, i.e., the number of elements

Available since 2.30

```
int size(complex_row_vector x)
```

The size of x, i.e., the number of elements

Available since 2.30

```
int size(matrix x)
```

The size of the matrix x. For example, if x is a 5×3 matrix, then `size(x)` is 15.

Available since 2.30

7.3. Complex matrix arithmetic operators

Stan supports all basic complex arithmetic operators using infix, prefix and postfix operations. This section lists the operations supported by Stan along with their argument and result types.

Negation prefix operators

```
complex_vector operator-(complex_vector x)
```

The negation of the vector x.

Available since 2.30

```
complex_row_vector operator-(complex_row_vector x)
```

The negation of the row vector x.

Available since 2.30

```
complex_matrix operator-(complex_matrix x)
```

The negation of the matrix x.

Available since 2.30

T operator-(T x)

Vectorized version of operator-. If T x is a (possibly nested) array of matrix types, $-x$ is the same shape array where each individual value is negated.

Available since 2.31

Infix complex_matrix operators

`complex_vector operator+(complex_vector x, complex_vector y)`

The sum of the vectors x and y .

Available since 2.30

`complex_row_vector operator+(complex_row_vector x, complex_row_vector y)`

The sum of the row vectors x and y .

Available since 2.30

`complex_matrix operator+(complex_matrix x, complex_matrix y)`

The sum of the matrices x and y .

Available since 2.30

`complex_vector operator-(complex_vector x, complex_vector y)`

The difference between the vectors x and y .

Available since 2.30

`complex_row_vector operator-(complex_row_vector x, complex_row_vector y)`

The difference between the row vectors x and y .

Available since 2.30

`complex_matrix operator-(complex_matrix x, complex_matrix y)`

The difference between the matrices x and y .

Available since 2.30

`complex_vector operator*(complex x, complex_vector y)`

The product of the scalar x and vector y .

Available since 2.30

`complex_row_vector operator*(complex x, complex_row_vector y)`

The product of the scalar x and the row vector y .

Available since 2.30

```
complex_matrix operator*(complex x, complex_matrix y)
```

The product of the scalar x and the matrix y

Available since 2.30

```
complex_vector operator*(complex_vector x, complex y)
```

The product of the scalar y and vector x

Available since 2.30

```
complex_matrix operator*(complex_vector x, complex_row_vector y)
```

The product of the vector x and row vector y

Available since 2.30

```
complex_row_vector operator*(complex_row_vector x, complex y)
```

The product of the scalar y and row vector x

Available since 2.30

```
complex operator*(complex_row_vector x, complex_vector y)
```

The product of the row vector x and vector y

Available since 2.30

```
complex_row_vector operator*(complex_row_vector x, complex_matrix y)
```

The product of the row vector x and matrix y

Available since 2.30

```
complex_matrix operator*(complex_matrix x, complex y)
```

The product of the scalar y and matrix x

Available since 2.30

```
complex_vector operator*(complex_matrix x, complex_vector y)
```

The product of the matrix x and vector y

Available since 2.30

```
complex_matrix operator*(complex_matrix x, complex_matrix y)
```

The product of the matrices x and y

Available since 2.30

Broadcast infix operators`complex_vector operator+(complex_vector x, complex y)`

The result of adding y to every entry in the vector x

Available since 2.30

`complex_vector operator+(complex x, complex_vector y)`

The result of adding x to every entry in the vector y

Available since 2.30

`complex_row_vector operator+(complex_row_vector x, complex y)`

The result of adding y to every entry in the row vector x

Available since 2.30

`complex_row_vector operator+(complex x, complex_row_vector y)`

The result of adding x to every entry in the row vector y

Available since 2.30

`complex_matrix operator+(complex_matrix x, complex y)`

The result of adding y to every entry in the matrix x

Available since 2.30

`complex_matrix operator+(complex x, complex_matrix y)`

The result of adding x to every entry in the matrix y

Available since 2.30

`complex_vector operator-(complex_vector x, complex y)`

The result of subtracting y from every entry in the vector x

Available since 2.30

`complex_vector operator-(complex x, complex_vector y)`

The result of adding x to every entry in the negation of the vector y

Available since 2.30

`complex_row_vector operator-(complex_row_vector x, complex y)`

The result of subtracting y from every entry in the row vector x

Available since 2.30

`complex_row_vector operator-(complex x, complex_row_vector y)`

The result of adding x to every entry in the negation of the row vector y

Available since 2.30

`complex_matrix operator-(complex_matrix x, complex y)`

The result of subtracting y from every entry in the matrix x

Available since 2.30

`complex_matrix operator+(complex x, complex_matrix y)`

The result of adding x to every entry in negation of the matrix y

Available since 2.30

`complex_vector operator/(complex_vector x, complex y)`

The result of dividing each entry in the vector x by y

Available since 2.30

`complex_row_vector operator/(complex_row_vector x, complex y)`

The result of dividing each entry in the row vector x by y

Available since 2.30

`complex_matrix operator/(complex_matrix x, complex y)`

The result of dividing each entry in the matrix x by y

Available since 2.30

7.4. Complex Transposition Operator

Complex `complex_matrix` transposition is represented using a postfix operator.

`complex_matrix operator'(complex_matrix x)`

The transpose of the matrix x, written as x'

Available since 2.30

`complex_row_vector operator'(complex_vector x)`

The transpose of the vector x, written as x'

Available since 2.30

`complex_vector operator'(complex_row_vector x)`

The transpose of the row vector x, written as x'

Available since 2.30

7.5. Complex elementwise functions

As in the real case, elementwise complex functions apply a function to each element of a vector or matrix, returning a result of the same shape as the argument.

`complex_vector operator.*(complex_vector x, complex_vector y)`

The elementwise product of x and y

Available since 2.30

`complex_row_vector`

`operator.*(complex_row_vector x, com-`

`plex_row_vector y)`

The elementwise product of x and y

Available since 2.30

`complex_matrix operator.*(complex_matrix x, complex_matrix y)`

The elementwise product of x and y

Available since 2.30

`complex_vector operator./(complex_vector x, complex_vector y)`

The elementwise quotient of x and y

Available since 2.30

`complex_vector operator./(complex x, complex_vector y)`

The elementwise quotient of x and y

Available since 2.30

`complex_vector operator./(complex_vector x, complex y)`

The elementwise quotient of x and y

Available since 2.30

`complex_row_vector`

`operator./(complex_row_vector x, com-`

`plex_row_vector y)`

The elementwise quotient of x and y

Available since 2.30

`complex_row_vector operator./(complex x, complex_row_vector y)`

The elementwise quotient of x and y

Available since 2.30

`complex_row_vector operator./(complex_row_vector x, complex y)`

The elementwise quotient of x and y

Available since 2.30

`complex_matrix operator./(complex_matrix x, complex_matrix y)`
The elementwise quotient of x and y

Available since 2.30

`complex_matrix operator./(complex x, complex_matrix y)`
The elementwise quotient of x and y

Available since 2.30

`complex_matrix operator./(complex_matrix x, complex y)`
The elementwise quotient of x and y

Available since 2.30

`vector operator.^(complex_vector x, complex_vector y)`
The elementwise power of y and x

Available since 2.30

`vector operator.^(complex_vector x, complex y)`
The elementwise power of y and x

Available since 2.30

`vector operator.^(complex x, complex_vector y)`
The elementwise power of y and x

Available since 2.30

`row_vector operator.^(complex_row_vector x, complex_row_vector y)`
The elementwise power of y and x

Available since 2.30

`row_vector operator.^(complex_row_vector x, complex y)`
The elementwise power of y and x

Available since 2.30

`row_vector operator.^(complex x, complex_row_vector y)`
The elementwise power of y and x

Available since 2.30

`matrix operator.^(complex_matrix x, complex_matrix y)`
The elementwise power of y and x

Available since 2.30

```
matrix operator.^( complex_matrix x, complex y)
```

The elementwise power of y and x

Available since 2.30

```
matrix operator.^(complex x, complex_matrix y)
```

The elementwise power of y and x

Available since 2.30

7.6. Dot products and specialized products for complex matrices

```
complex dot_product(complex_vector x, complex_vector y)
```

The dot product of x and y

Available since 2.30

```
complex dot_product(complex_vector x, complex_row_vector y)
```

The dot product of x and y

Available since 2.30

```
complex dot_product(complex_row_vector x, complex_vector y)
```

The dot product of x and y

Available since 2.30

```
complex dot_product(complex_row_vector x, complex_row_vector y)
```

The dot product of x and y

Available since 2.30

```
complex_row_vector      columns_dot_product(complex_vector x, complex_vector y)
```

The dot product of the columns of x and y

Available since 2.30

```
complex_row_vector      columns_dot_product(complex_row_vector x, complex_row_vector y)
```

The dot product of the columns of x and y

Available since 2.30

```
complex_row_vector      columns_dot_product(complex_matrix x, complex_matrix y)
```

The dot product of the columns of x and y

Available since 2.30

`complex_vector rows_dot_product(complex_vector x, complex_vector y)`

The dot product of the rows of x and y

Available since 2.30

`complex_vector rows_dot_product(complex_row_vector x, complex_row_vector y)`

The dot product of the rows of x and y

Available since 2.30

`complex_vector rows_dot_product(complex_matrix x, complex_matrix y)`

The dot product of the rows of x and y

Available since 2.30

`complex dot_self(complex_vector x)`

The dot product of the vector x with itself

Available since 2.30

`complex dot_self(complex_row_vector x)`

The dot product of the row vector x with itself

Available since 2.30

`complex_row_vector columns_dot_self(complex_vector x)`

The dot product of the columns of x with themselves

Available since 2.30

`complex_row_vector columns_dot_self(complex_row_vector x)`

The dot product of the columns of x with themselves

Available since 2.30

`complex_row_vector columns_dot_self(complex_matrix x)`

The dot product of the columns of x with themselves

Available since 2.30

`complex_vector rows_dot_self(complex_vector x)`

The dot product of the rows of x with themselves

Available since 2.30

`complex_vector rows_dot_self(complex_row_vector x)`

The dot product of the rows of x with themselves

Available since 2.30

`complex_vector rows_dot_self(complex_matrix x)`

The dot product of the rows of x with themselves

Available since 2.30

Specialized products

`complex_matrix diag_pre_multiply(complex_vector v, complex_matrix m)`

Return the product of the diagonal matrix formed from the vector v and the matrix m, i.e., `diag_matrix(v) * m`.

Available since 2.30

`complex_matrix diag_pre_multiply(complex_row_vector v, complex_matrix m)`

Return the product of the diagonal matrix formed from the vector rv and the matrix m, i.e., `diag_matrix(rv) * m`.

Available since 2.30

`complex_matrix diag_post_multiply(complex_matrix m, complex_vector v)`

Return the product of the matrix m and the diagonal matrix formed from the vector v, i.e., `m * diag_matrix(v)`.

Available since 2.30

`complex_matrix diag_post_multiply(complex_matrix m, complex_row_vector v)`

Return the product of the matrix m and the diagonal matrix formed from the the row vector rv, i.e., `m * diag_matrix(rv)`.

Available since 2.30

7.7. Complex reductions

Sums and products

`complex sum(complex_vector x)`

The sum of the values in x, or 0 if x is empty

Available since 2.30

`complex sum(complex_row_vector x)`

The sum of the values in `x`, or 0 if `x` is empty

Available since 2.30

`complex sum(complex_matrix x)`

The sum of the values in `x`, or 0 if `x` is empty

Available since 2.30

`complex prod(complex_vector x)`

The product of the values in `x`, or 1 if `x` is empty

Available since 2.30

`complex prod(complex_row_vector x)`

The product of the values in `x`, or 1 if `x` is empty

Available since 2.30

`complex prod(complex_matrix x)`

The product of the values in `x`, or 1 if `x` is empty

Available since 2.30

7.8. Vectorized accessor functions

Much like with complex scalars, two functions are defined to get the real and imaginary components of complex-valued objects.

Type “demotion”

These functions return the same shape (e.g., matrix, vector, row vector, or array) object as their input, but demoted to a `real` type. For example, `get_real(complex_matrix M)` yields a matrix containing the real component of each value in `M`.

The following table contains examples of what this notation can mean:

Type <code>T</code>	Type <code>T_demoted</code>
<code>complex</code>	<code>real</code>
<code>complex_vector</code>	<code>vector</code>
<code>complex_row_vector</code>	<code>row_vector</code>
<code>complex_matrix</code>	<code>matrix</code>
<code>array[] complex</code>	<code>array[] real</code>
<code>array[,,] complex</code>	<code>array[,,] real</code>

Real and imaginary component accessor functions

`T_demoted get_real(T x)`

Given an object of complex type `T`, return the same shape object but of type `real` by getting the real component of each element of `x`.

Available since 2.30

`T_demoted get_imag(T x)`

Given an object of complex type `T`, return the same shape object but of type `real` by getting the imaginary component of each element of `x`.

Available since 2.30

For example, given the Stan declaration

```
complex_vector[2] z = [3+4i, 5+6i]';
```

A call `get_real(z)` will yield the vector `[3, 5]'`, and a call `get_imag(z)` will yield the vector `[4, 6]'`.

7.9. Complex broadcast functions

The following broadcast functions allow vectors, row vectors and matrices to be created by copying a single element into all of their cells. Matrices may also be created by stacking copies of row vectors vertically or stacking copies of column vectors horizontally.

`complex_vector rep_vector(complex z, int m)`

Return the size `m` (column) vector consisting of copies of `z`.

Available since 2.30

`complex_row_vector rep_row_vector(complex z, int n)`

Return the size `n` row vector consisting of copies of `z`.

Available since 2.30

`complex_matrix rep_matrix(complex z, int m, int n)`

Return the `m` by `n` matrix consisting of copies of `z`.

Available since 2.30

`complex_matrix rep_matrix(complex_vector v, int n)`

Return the `m` by `n` matrix consisting of `n` copies of the (column) vector `v` of size `m`.

Available since 2.30

```
complex_matrix rep_matrix(complex_row_vector rv, int m)
```

Return the $m \times n$ matrix consisting of m copies of the row vector rv of size n .

Available since 2.30

Symmetrization

```
complex_matrix symmetrize_from_lower_tri(complex_matrix A)
```

Construct a symmetric matrix from the lower triangle of A .

Available since 2.30

7.10. Diagonal complex matrix functions

```
complex_matrix add_diag(complex_matrix m, complex_row_vector d)
```

Add row_vector d to the diagonal of matrix m .

Available since 2.30

```
complex_matrix add_diag(complex_matrix m, complex_vector d)
```

Add vector d to the diagonal of matrix m .

Available since 2.30

```
complex_matrix add_diag(complex_matrix m, complex d)
```

Add scalar d to every diagonal element of matrix m .

Available since 2.30

```
complex_vector diagonal(complex_matrix x)
```

The diagonal of the matrix x

Available since 2.30

```
complex_matrix diag_matrix(complex_vector x)
```

The diagonal matrix with diagonal x

Available since 2.30

7.11. Slicing and blocking functions for complex matrices

Stan provides several functions for generating slices or blocks or diagonal entries for matrices.

Columns and rows

```
complex_vector col(complex_matrix x, int n)
```

The n -th column of matrix x

Available since 2.30

`complex_row_vector row(complex_matrix x, int m)`

The m-th row of matrix x

Available since 2.30

Block operations

Matrix slicing operations

`complex_matrix block(complex_matrix x, int i, int j, int n_rows, int n_cols)`

Return the submatrix of x that starts at row i and column j and extends n_rows rows and n_cols columns.

Available since 2.30

`complex_vector sub_col(complex_matrix x, int i, int j, int n_rows)`

Return the sub-column of x that starts at row i and column j and extends n_rows rows and 1 column.

Available since 2.30

`complex_row_vector sub_row(complex_matrix x, int i, int j, int n_cols)`

Return the sub-row of x that starts at row i and column j and extends 1 row and n_cols columns.

Available since 2.30

Vector slicing operations.

`complex_vector head(complex_vector v, int n)`

Return the vector consisting of the first n elements of v.

Available since 2.30

`complex_row_vector head(complex_row_vector rv, int n)`

Return the row vector consisting of the first n elements of rv.

Available since 2.30

`complex_vector tail(complex_vector v, int n)`

Return the vector consisting of the last n elements of v.

Available since 2.30

`complex_row_vector tail(complex_row_vector rv, int n)`

Return the row vector consisting of the last n elements of rv.

Available since 2.30

```
complex_vector segment(complex_vector v, int i, int n)
```

Return the vector consisting of the n elements of v starting at i; i.e., elements i through through i + n - 1.

Available since 2.30

```
complex_row_vector segment(complex_row_vector rv, int i, int n)
```

Return the row vector consisting of the n elements of rv starting at i; i.e., elements i through through i + n - 1.

Available since 2.30

7.12. Complex matrix concatenation

Horizontal concatenation

```
complex_matrix append_col(complex_matrix x, complex_matrix y)
```

Combine matrices x and y by column. The matrices must have the same number of rows.

Available since 2.30

```
complex_matrix append_col(complex_matrix x, complex_vector y)
```

Combine matrix x and vector y by column. The matrix and the vector must have the same number of rows.

Available since 2.30

```
complex_matrix append_col(complex_vector x, complex_matrix y)
```

Combine vector x and matrix y by column. The vector and the matrix must have the same number of rows.

Available since 2.30

```
complex_matrix append_col(complex_vector x, complex_vector y)
```

Combine vectors x and y by column. The vectors must have the same number of rows.

Available since 2.30

```
complex_row_vector  
complex_row_vector y)
```

```
append_col(complex_row_vector x, com-
```

Combine row vectors x and y (of any size) into another row vector by appending y to the end of x.

Available since 2.30

```
complex_row_vector append_col(complex x, complex_row_vector y)
```

Append x to the front of y, returning another row vector.

Available since 2.30

```
complex_row_vector append_col(complex_row_vector x, complex y)
```

Append y to the end of x, returning another row vector.

Available since 2.30

Vertical concatenation

```
complex_matrix append_row(complex_matrix x, complex_matrix y)
```

Combine matrices x and y by row. The matrices must have the same number of columns.

Available since 2.30

```
complex_matrix append_row(complex_matrix x, complex_row_vector y)
```

Combine matrix x and row vector y by row. The matrix and the row vector must have the same number of columns.

Available since 2.30

```
complex_matrix append_row(complex_row_vector x, complex_matrix y)
```

Combine row vector x and matrix y by row. The row vector and the matrix must have the same number of columns.

Available since 2.30

```
complex_matrix append_row(complex_row_vector x, complex_row_vector y)
```

Combine row vectors x and y by row. The row vectors must have the same number of columns.

Available since 2.30

```
complex_vector append_row(complex_vector x, complex_vector y)
```

Concatenate vectors x and y of any size into another vector.

Available since 2.30

```
complex_vector append_row(complex x, complex_vector y)
```

Append x to the top of y, returning another vector.

Available since 2.30

`complex_vector append_row(complex_vector x, complex y)`

Append y to the bottom of x , returning another vector.

Available since 2.30

7.13. Complex special matrix functions

Fast Fourier transforms

Stan's fast Fourier transform functions take the standard definition of the discrete Fourier transform (see the definitions below for specifics) and scale the inverse transform by one over dimensionality so that the following identities hold for complex vectors u and v ,

$$\text{fft}(\text{inv_fft}(u)) == u \quad \text{inv_fft}(\text{fft}(v)) == v$$

and in the 2-dimensional case for complex matrices A and B ,

$$\text{fft2}(\text{inv_fft2}(A)) == A \quad \text{inv_fft2}(\text{fft2}(B)) == B$$

Although the FFT functions only accept complex inputs, real vectors and matrices will be promoted to their complex counterparts before applying the FFT functions.

`complex_vector fft(complex_vector v)`

Return the discrete Fourier transform of the specified complex vector v . If $v \in \mathbb{C}^N$ is a complex vector with N elements and $u = \text{fft}(v)$, then

$$u_n = \sum_{m < n} v_m \cdot \exp\left(\frac{-n \cdot m \cdot 2 \cdot \pi \cdot \sqrt{-1}}{N}\right).$$

Available since 2.30

`complex_matrix fft2(complex_matrix m)`

Return the 2D discrete Fourier transform of the specified complex matrix m . The 2D FFT is defined as the result of applying the FFT to each row and then to each column.

Available since 2.30

`complex_vector inv_fft(complex_vector u)`

Return the inverse of the discrete Fourier transform of the specified complex vector u . The inverse FFT (this function) is scaled so that $\text{fft}(\text{inv_fft}(u)) == u$. If $u \in \mathbb{C}^N$ is a complex vector with N elements and $v = \text{fft}^{-1}(u)$, then

$$v_n = \frac{1}{N} \sum_{m < n} u_m \cdot \exp\left(\frac{n \cdot m \cdot 2 \cdot \pi \cdot \sqrt{-1}}{N}\right).$$

This only differs from the FFT by the sign inside the exponential and the scaling. The $\frac{1}{N}$ scaling ensures that `fft(inv_fft(u)) == u` and `inv_fft(fft(v)) == v` for complex vectors `u` and `v`.

Available since 2.30

`complex_matrix inv_fft2(complex_matrix m)`

Return the inverse of the 2D discrete Fourier transform of the specified complex matrix `m`. The 2D inverse FFT is defined as the result of applying the inverse FFT to each row and then to each column. The invertible scaling of the inverse FFT ensures `fft2(inv_fft2(A)) == A` and `inv_fft2(fft2(B)) == B`.

Available since 2.30

Cumulative sums

The cumulative sum of a sequence x_1, \dots, x_N is the sequence y_1, \dots, y_N , where

$$y_n = \sum_{m=1}^n x_m.$$

`array[] complex cumulative_sum(array[] complex x)`

The cumulative sum of `x`

Available since 2.30

`complex_vector cumulative_sum(complex_vector v)`

The cumulative sum of `v`

Available since 2.30

`complex_row_vector cumulative_sum(complex_row_vector rv)`

The cumulative sum of `rv`

Available since 2.30

7.14. Complex linear algebra functions

Complex matrix division operators and functions

In general, it is much more efficient and also more arithmetically stable to use matrix division than to multiply by an inverse.

Complex matrix division operators

`complex_row_vector operator/(complex_row_vector b, complex_matrix A)`

The right division of `b` by `A`; equivalently `b * inverse(A)`

Available since 2.30

`complex_matrix operator/(complex_matrix B, complex_matrix A)`

The right division of B by A; equivalently `B * inverse(A)`

Available since 2.30

Linear algebra functions

Trace

`complex trace(complex_matrix A)`

The trace of A, or 0 if A is empty; A is not required to be diagonal

Available since 2.30

Eigendecomposition

`complex_vector eigenvalues(complex_matrix A)`

The complex-valued vector of eigenvalues of the matrix A. The eigenvalues are repeated according to their algebraic multiplicity, so there are as many eigenvalues as rows in the matrix. The eigenvalues are not sorted in any particular order.

Available since 2.32

`complex_matrix eigenvectors(complex_matrix A)`

The matrix with the complex-valued (column) eigenvectors of the matrix A in the same order as returned by the function `eigenvalues`

Available since 2.32

`tuple(complex_matrix, complex_vector) eigendecompose(complex_matrix A)`

Return the matrix of (column) eigenvectors and vector of eigenvalues of the matrix A. This function is equivalent to `(eigenvectors(A), eigenvalues(A))` but with a lower computational cost due to the shared work between the two results.

Available since 2.33

`complex_vector eigenvalues_sym(complex_matrix A)`

The vector of eigenvalues of a symmetric matrix A in ascending order

Available since 2.30

`complex_matrix eigenvectors_sym(complex_matrix A)`

The matrix with the (column) eigenvectors of symmetric matrix A in the same order as returned by the function `eigenvalues_sym`

Available since 2.30

tuple(complex_matrix, complex_vector) **eigendecom-**
pose_sym(complex_matrix A)

Return the matrix of (column) eigenvectors and vector of eigenvalues of the symmetric matrix A. This function is equivalent to `(eigenvectors_sym(A), eigenvalues_sym(A))` but with a lower computational cost due to the shared work between the two results.

Available since 2.33

Because multiplying an eigenvector by -1 results in an eigenvector, eigenvectors returned by a decomposition are only identified up to a sign change. In order to compare the eigenvectors produced by Stan's eigendecomposition to others, signs may need to be normalized in some way, such as by fixing the sign of a component, or doing comparisons allowing a multiplication by -1 .

The condition number of a symmetric matrix is defined to be the ratio of the largest eigenvalue to the smallest eigenvalue. Large condition numbers lead to difficulty in numerical algorithms such as computing inverses, and thus known as "ill conditioned." The ratio can even be infinite in the case of singular matrices (i.e., those with eigenvalues of 0).

Singular value decomposition

The matrix A can be decomposed into a diagonal matrix of singular values, D, and matrices of its left and right singular vectors, U and V,

$$A = UDV^T.$$

The matrices of singular vectors here are thin. That is for an N by P input A, $M = \min(N, P)$, U is size N by M and V is size P by M .

vector singular_values(complex_matrix A)

The singular values of A in descending order

Available since 2.30

complex_matrix svd_U(complex_matrix A)

The left-singular vectors of A

Available since 2.30

complex_matrix svd_V(complex_matrix A)

The right-singular vectors of A

Available since 2.30

tuple(complex_matrix, vector, complex_matrix) `svd`(complex_matrix A)
 Returns a tuple containing the left-singular vectors of A, the singular values of A in descending order, and the right-singular values of A. This function is equivalent to (`svd_U(A)`, `singular_values(A)`, `svd_V(A)`) but with a lower computational cost due to the shared work between the different components.

Available since 2.33

Complex Schur Decomposition

The complex Schur decomposition of a square matrix A produces a complex unitary matrix U and a complex upper-triangular Schur form matrix T such that

$$A = U \cdot T \cdot U^{-1}$$

Since U is unitary, its inverse is also its conjugate transpose, $U^{-1} = U^*$, $U^*(i, j) = \text{conj}(U(j, i))$

complex_matrix `complex_schur_decompose_t`(matrix A)

Compute the upper-triangular Schur form matrix of the complex Schur decomposition of A.

Available since 2.31

complex_matrix `complex_schur_decompose_t`(complex_matrix A)

Compute the upper-triangular Schur form matrix of the complex Schur decomposition of A.

Available since 2.31

complex_matrix `complex_schur_decompose_u`(matrix A)

Compute the unitary matrix of the complex Schur decomposition of A.

Available since 2.31

complex_matrix `complex_schur_decompose_u`(complex_matrix A)

Compute the unitary matrix of the complex Schur decomposition of A.

Available since 2.31

tuple(complex_matrix, complex_matrix) `complex_schur_decompose`(matrix A)

Returns the unitary matrix and the upper-triangular Schur form matrix of the complex Schur decomposition of A. This function is equivalent to (`complex_schur_decompose_u(A)`, `complex_schur_decompose_t(A)`) but with a lower computational cost due to the shared work between the two results. This

overload is equivalent to `complex_schur_decompose(to_complex(A,0))` but is more efficient.

Available since 2.33

`tuple(complex_matrix, complex_matrix) complex_schur_decompose(complex_matrix A)`

Returns the unitary matrix and the upper-triangular Schur form matrix of the complex Schur decomposition of A. This function is equivalent to `(complex_schur_decompose_u(A), complex_schur_decompose_t(A))` but with a lower computational cost due to the shared work between the two results.

Available since 2.33

7.15. Reverse functions for complex matrices

`complex_vector reverse(complex_vector v)`

Return a new vector containing the elements of the argument in reverse order.

Available since 2.30

`complex_row_vector reverse(complex_row_vector v)`

Return a new row vector containing the elements of the argument in reverse order.

Available since 2.30

8. Sparse Matrix Operations

For sparse matrices, for which many elements are zero, it is more efficient to use specialized representations to save memory and speed up matrix arithmetic (including derivative calculations). Given Stan's implementation, there is substantial space (memory) savings by using sparse matrices. Because of the ease of optimizing dense matrix operations, speed improvements only arise at 90% or even greater sparsity; below that level, dense matrices are faster but use more memory.

Because of this speedup and space savings, it may even be useful to read in a dense matrix and convert it to a sparse matrix before multiplying it by a vector. This chapter covers a very specific form of sparsity consisting of a sparse matrix multiplied by a dense vector.

8.1. Compressed row storage

Sparse matrices are represented in Stan using compressed row storage (CSR). For example, the matrix

$$A = \begin{bmatrix} 19 & 27 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 52 \\ 81 & 0 & 95 & 33 \end{bmatrix}$$

is translated into a vector of the non-zero real values, read by row from the matrix A ,

$$w(A) = [19 \ 27 \ 52 \ 81 \ 95 \ 33]^\top,$$

an array of integer column indices for the values,

$$v(A) = [1 \ 2 \ 4 \ 1 \ 3 \ 4],$$

and an array of integer indices indicating where in $w(A)$ a given row's values start,

$$u(A) = [1 \ 3 \ 3 \ 4 \ 7],$$

with a padded value at the end to guarantee that

$$u(A)[n+1] - u(A)[n]$$

is the number of non-zero elements in row n of the matrix (here 2, 0, 1, and 3). Note that because the second row has no non-zero elements both the second and third

elements of $u(A)$ correspond to the third element of $w(A)$, which is 52. The values $(w(A), v(A), u(A))$ are sufficient to reconstruct A .

The values are structured so that there is a real value and integer column index for each non-zero entry in the array, plus one integer for each row of the matrix, plus one for padding. There is also underlying storage for internal container pointers and sizes. The total memory usage is roughly $12K + M$ bytes plus a small constant overhead, which is often considerably fewer bytes than the $M \times N$ required to store a dense matrix. Even more importantly, zero values do not introduce derivatives under multiplication or addition, so many storage and evaluation steps are saved when sparse matrices are multiplied.

8.2. Conversion functions

Conversion functions between dense and sparse matrices are provided.

Dense to sparse conversion

Converting a dense matrix m to a sparse representation produces a vector w and two integer arrays, u and v .

`vector csr_extract_w(matrix a)`

Return non-zero values in matrix a ; see section [compressed row storage](#).

Available since 2.8

`array[] int csr_extract_v(matrix a)`

Return column indices for values in `csr_extract_w(a)`; see [compressed row storage](#).

Available since 2.8

`array[] int csr_extract_u(matrix a)`

Return array of row starting indices for entries in `csr_extract_w(a)` followed by the size of `csr_extract_w(a)` plus one; see section [compressed row storage](#).

Available since 2.8

`tuple(vector, array[] int, array[] int) csr_extract(matrix a)`

Return all three components of the CSR representation of the matrix a ; see section [compressed row storage](#). This function is equivalent to `(csr_extract_w(a), csr_extract_v(a), csr_extract_u(a))`.

Available since 2.33

Sparse to dense conversion

To convert a sparse matrix representation to a dense matrix, there is a single function.

```
matrix  csr_to_dense_matrix(int m, int n, vector w, array[] int v,
array[] int u)
```

Return dense $m \times n$ matrix with non-zero matrix entries w , column indices v , and row starting indices u ; the vector w and array v must be the same size (corresponding to the total number of nonzero entries in the matrix), array v must have index values bounded by m , array u must have length equal to $m + 1$ and contain index values bounded by the number of nonzeros (except for the last entry, which must be equal to the number of nonzeros plus one). See section [compressed row storage](#) for more details.

Available since 2.10

8.3. Sparse matrix arithmetic

Sparse matrix multiplication

The only supported operation is the multiplication of a sparse matrix A and a dense vector b to produce a dense vector $A b$. Multiplying a dense row vector b and a sparse matrix A can be coded using transposition as

$$b A = (A^\top b^\top)^\top,$$

but care must be taken to represent A^\top rather than A as a sparse matrix.

```
vector  csr_matrix_times_vector(int m, int n, vector w, array[] int v,
array[] int u, vector b)
```

Multiply the $m \times n$ matrix represented by values w , column indices v , and row start indices u by the vector b ; see [compressed row storage](#).

Available since 2.18

9. Mixed Operations

These functions perform conversions between Stan containers matrix, vector, row vector and arrays.

`matrix to_matrix(matrix m)`

Return the matrix `m` itself.

Available since 2.3

`complex_matrix to_matrix(complex_matrix m)`

Return the matrix `m` itself.

Available since 2.30

`matrix to_matrix(vector v)`

Convert the column vector `v` to a `size(v)` by 1 matrix.

Available since 2.3

`complex_matrix to_matrix(complex_vector v)`

Convert the column vector `v` to a `size(v)` by 1 matrix.

Available since 2.30

`matrix to_matrix(row_vector v)`

Convert the row vector `v` to a 1 by `size(v)` matrix.

Available since 2.3

`complex_matrix to_matrix(complex_row_vector v)`

Convert the row vector `v` to a 1 by `size(v)` matrix.

Available since 2.30

`matrix to_matrix(matrix M, int m, int n)`

Convert a matrix `A` to a matrix with `m` rows and `n` columns filled in column-major order.

Available since 2.15

`complex_matrix to_matrix(complex_matrix M, int m, int n)`

Convert a matrix `A` to a matrix with `m` rows and `n` columns filled in column-major order.

Available since 2.30

`matrix to_matrix(vector v, int m, int n)`

Convert a vector v to a matrix with m rows and n columns filled in column-major order.

Available since 2.15

`complex_matrix to_matrix(complex_vector v, int m, int n)`

Convert a vector v to a matrix with m rows and n columns filled in column-major order.

Available since 2.30

`matrix to_matrix(row_vector v, int m, int n)`

Convert a row_vector v to a matrix with m rows and n columns filled in column-major order.

Available since 2.15

`complex_matrix to_matrix(complex_row_vector v, int m, int n)`

Convert a row vector v to a matrix with m rows and n columns filled in column-major order.

Available since 2.30

`matrix to_matrix(matrix A, int m, int n, int col_major)`

Convert a matrix A to a matrix with m rows and n columns filled in row-major order if col_major equals 0 (otherwise, they get filled in column-major order).

Available since 2.15

`complex_matrix to_matrix(complex_matrix A, int m, int n, int col_major)`

Convert a matrix A to a matrix with m rows and n columns filled in row-major order if col_major equals 0 (otherwise, they get filled in column-major order).

Available since 2.30

`matrix to_matrix(vector v, int m, int n, int col_major)`

Convert a vector v to a matrix with m rows and n columns filled in row-major order if col_major equals 0 (otherwise, they get filled in column-major order).

Available since 2.15

`complex_matrix to_matrix(complex_vector v, int m, int n, int col_major)`

Convert a vector v to a matrix with m rows and n columns filled in row-major order if `col_major` equals 0 (otherwise, they get filled in column-major order).

Available since 2.30

```
matrix to_matrix(row_vector v, int m, int n, int col_major)
```

Convert a row vector v to a matrix with m rows and n columns filled in row-major order if `col_major` equals 0 (otherwise, they get filled in column-major order).

Available since 2.15

```
complex_matrix to_matrix(complex_row_vector v, int m, int n, int col_major)
```

Convert a row vector v to a matrix with m rows and n columns filled in row-major order if `col_major` equals 0 (otherwise, they get filled in column-major order).

Available since 2.30

```
matrix to_matrix(array[] real a, int m, int n)
```

Convert a one-dimensional array a to a matrix with m rows and n columns filled in column-major order.

Available since 2.15

```
matrix to_matrix(array[] int a, int m, int n)
```

Convert a one-dimensional array a to a matrix with m rows and n columns filled in column-major order.

Available since 2.15

```
complex_matrix to_matrix(array[] complex a, int m, int n)
```

Convert a one-dimensional array a to a matrix with m rows and n columns filled in column-major order.

Available since 2.30

```
matrix to_matrix(array[] real a, int m, int n, int col_major)
```

Convert a one-dimensional array a to a matrix with m rows and n columns filled in row-major order if `col_major` equals 0 (otherwise, they get filled in column-major order).

Available since 2.15

```
matrix to_matrix(array[] int a, int m, int n, int col_major)
```

Convert a one-dimensional array a to a matrix with m rows and n columns filled in

row-major order if `col_major` equals 0 (otherwise, they get filled in column-major order).

Available since 2.15

`complex_matrix to_matrix(array[] complex a, int m, int n, int col_major)`

Convert a one-dimensional array `a` to a matrix with `m` rows and `n` columns filled in row-major order if `col_major` equals 0 (otherwise, they get filled in column-major order).

Available since 2.30

`matrix to_matrix(array[] row_vector vs)`

Convert a one-dimensional array of row vectors to a matrix, where the size of the array is the number of rows of the resulting matrix and the length of row vectors is the number of columns.

Available since 2.28

`complex_matrix to_matrix(array[] complex_row_vector vs)`

Convert a one-dimensional array of row vectors to a matrix, where the size of the array is the number of rows of the resulting matrix and the length of row vectors is the number of columns.

Available since 2.30

`matrix to_matrix(array[,] real a)`

Convert the two dimensional array `a` to a matrix with the same dimensions and indexing order.

Available since 2.3

`matrix to_matrix(array[,] int a)`

Convert the two dimensional array `a` to a matrix with the same dimensions and indexing order. If any of the dimensions of `a` are zero, the result will be a 0×0 matrix.

Available since 2.3

`complex_matrix to_matrix(array[,] complex a)`

Convert the two dimensional array `a` to a matrix with the same dimensions and indexing order.

Available since 2.30

```
vector to_vector(matrix m)
```

Convert the matrix *m* to a column vector in column-major order.

Available since 2.0

```
complex_vector to_vector(complex_matrix m)
```

Convert the matrix *m* to a column vector in column-major order.

Available since 2.30

```
vector to_vector(vector v)
```

Return the column vector *v* itself.

Available since 2.3

```
complex_vector to_vector(complex_vector v)
```

Return the column vector *v* itself.

Available since 2.30

```
vector to_vector(row_vector v)
```

Convert the row vector *v* to a column vector.

Available since 2.3

```
complex_vector to_vector(complex_row_vector v)
```

Convert the row vector *v* to a column vector.

Available since 2.30

```
vector to_vector(array[] real a)
```

Convert the one-dimensional array *a* to a column vector.

Available since 2.3

```
vector to_vector(array[] int a)
```

Convert the one-dimensional integer array *a* to a column vector.

Available since 2.3

```
complex_vector to_vector(array[] complex a)
```

Convert the one-dimensional complex array *a* to a column vector.

Available since 2.30

```
row_vector to_row_vector(matrix m)
```

Convert the matrix *m* to a row vector in column-major order.

Available since 2.3

```
complex_row_vector to_row_vector(complex_matrix m)
```

Convert the matrix m to a row vector in column-major order.

Available since 2.30

```
row_vector to_row_vector(vector v)
```

Convert the column vector v to a row vector.

Available since 2.3

```
complex_row_vector to_row_vector(complex_vector v)
```

Convert the column vector v to a row vector.

Available since 2.30

```
row_vector to_row_vector(row_vector v)
```

Return the row vector v itself.

Available since 2.3

```
complex_row_vector to_row_vector(complex_row_vector v)
```

Return the row vector v itself.

Available since 2.30

```
row_vector to_row_vector(array[] real a)
```

Convert the one-dimensional array a to a row vector.

Available since 2.3

```
row_vector to_row_vector(array[] int a)
```

Convert the one-dimensional array a to a row vector.

Available since 2.3

```
complex_row_vector to_row_vector(array[] complex a)
```

Convert the one-dimensional complex array a to a row vector.

Available since 2.30

```
array[,] real to_array_2d(matrix m)
```

Convert the matrix m to a two dimensional array with the same dimensions and indexing order.

Available since 2.3

```
array[,] complex to_array_2d(complex_matrix m)
```

Convert the matrix m to a two dimensional array with the same dimensions and

indexing order.

Available since 2.30

`array[] real to_array_1d(vector v)`

Convert the column vector v to a one-dimensional array.

Available since 2.3

`array[] complex to_array_1d(complex_vector v)`

Convert the column vector v to a one-dimensional array.

Available since 2.30

`array[] real to_array_1d(row_vector v)`

Convert the row vector v to a one-dimensional array.

Available since 2.3

`array[] complex to_array_1d(complex_row_vector v)`

Convert the row vector v to a one-dimensional array.

Available since 2.30

`array[] real to_array_1d(matrix m)`

Convert the matrix m to a one-dimensional array in column-major order.

Available since 2.3

`array[] real to_array_1d(complex_matrix m)`

Convert the matrix m to a one-dimensional array in column-major order.

Available since 2.30

`array[] real to_array_1d(array[...] real a)`

Convert the array a (of any dimension up to 10) to a one-dimensional array in row-major order.

Available since 2.3

`array[] int to_array_1d(array[...] int a)`

Convert the array a (of any dimension up to 10) to a one-dimensional array in row-major order.

Available since 2.3

`array[] complex to_array_1d(array[...] complex a)`

Convert the array a (of any dimension up to 10) to a one-dimensional array in

row-major order.

Available since 2.30

10. Compound Arithmetic and Assignment

Compound arithmetic and assignment statements combine an arithmetic operation and assignment, replacing a statement such as

```
x = x op y;
```

with the more compact compound form

```
x op= y;
```

For example, $x = x + 1$; may be replaced with $x += 1$;. This works for all types that support arithmetic, including the scalar types `int`, `real`, `complex`, the real matrix types `vector`, `row_vector`, and `matrix`, and the complex matrix types, `complex_vector`, `complex_row_vector`, and `complex_matrix`.

10.1. Compound addition and assignment

Compound addition and assignment works wherever the corresponding addition and assignment would be well formed.

```
void operator+=(T x, U y)
```

$x += y$ is equivalent to $x = x + y$. Defined for all types `T` and `U` where $T = T + U$ is well formed.

Available since 2.17, complex signatures added in 2.30

10.2. Compound subtraction and assignment

Compound addition and assignment works wherever the corresponding subtraction and assignment would be well formed.

```
void operator-=(T x, U y)
```

$x -= y$ is equivalent to $x = x - y$. Defined for all types `T` and `U` where $T = T - U$ is well formed.

Available since 2.17, complex signatures added in 2.30

10.3. Compound multiplication and assignment

Compound multiplication and assignment works wherever the corresponding multiplication and assignment would be well formed.

```
void operator*=(T x, U y)
```

$x *= y$ is equivalent to $x = x * y$. Defined for all types T and U where $T = T * U$ is well formed.

Available since 2.17, complex signatures added in 2.30

10.4. Compound division and assignment

Compound division and assignment works wherever the corresponding division and assignment would be well formed.

```
void operator/=(T x, U y)
```

$x /= y$ is equivalent to $x = x / y$. Defined for all types T and U where $T = T / U$ is well formed.

Available since 2.17, complex signatures added in 2.30

10.5. Compound elementwise multiplication and assignment

Compound elementwise multiplication and assignment works wherever the corresponding multiplication and assignment would be well formed.

```
void operator.*=(T x, U y)
```

$x .*= y$ is equivalent to $x = x .* y$. Defined for all types T and U where $T = T .* U$ is well formed.

Available since 2.17, complex signatures added in 2.30

10.6. Compound elementwise division and assignment

Compound elementwise division and assignment works wherever the corresponding division and assignment would be well formed.

```
void operator./=(T x, U y)
```

$x ./= y$ is equivalent to $x = x ./ y$. Defined for all types T and U where $T = T ./ U$ is well formed.

Available since 2.17, complex signatures added in 2.30

11. Higher-Order Functions

Stan provides a few higher-order functions that act on other functions. In all cases, the function arguments to the higher-order functions are defined as functions within the Stan language and passed by name to the higher-order functions.

11.1. Algebraic equation solvers

Stan provides two built-in algebraic equation solvers, respectively based on the Newton method and the Powell “dog leg” hybrid method. Empirically the Newton method is found to be faster and its use is recommended for most problems.

An algebraic solver is a higher-order function, i.e. it takes another function as one of its arguments. Other functions in Stan which share this feature are the differential equation solvers (see section [Ordinary Differential Equation \(ODE\) Solvers](#) and [Differential Algebraic Equation \(DAE\) solver](#)). Ordinary Stan functions do not allow functions as arguments.

Specifying an algebraic equation as a function

An algebraic system is specified as an ordinary function in Stan within the function block. The function must return a vector and takes in, as its first argument, the unknowns y we wish to solve for, also passed as a vector. This argument is followed by additional arguments as specified by the user; we call such arguments *variadic arguments* and denote them \dots . The signature of the algebraic system is then:

```
vector algebra_system (vector y, ...)
```

There is no type restriction for the variadic arguments and each argument can be passed as data or parameter. However users should use parameter arguments only when necessary and mark data arguments with the keyword `data`. In the below example, the last variadic argument, x , is restricted to being data:

```
vector algebra_system (vector y, vector theta, data vector x)
```

Distinguishing data and parameter is important for computational reasons. Augmenting the total number of parameters increases the cost of propagating derivatives through the solution to the algebraic equation, and ultimately the computational cost of evaluating the gradients.

Call to the algebraic solver

`vector solve_newton(function algebra_system, vector y_guess, ...)`
 Solves the algebraic system, given an initial guess, using Newton's method.

Available since 2.31

`vector solve_newton_tol(function algebra_system, vector y_guess, data real scaling_step, data real f_tol, int max_steps, ...)`
 Solves the algebraic system, given an initial guess, using Newton's method with additional control parameters for the solver.

Available since 2.31

`vector solve_powell(function algebra_system, vector y_guess, ...)`
 Solves the algebraic system, given an initial guess, using Powell's hybrid method.

Available since 2.31

`vector solve_powell_tol(function algebra_system, vector y_guess, data real rel_tol, data real f_tol, int max_steps, ...)`
 Solves the algebraic system, given an initial guess, using Powell's hybrid method with additional control parameters for the solver.

Available since 2.31

Arguments to the algebraic solver

The arguments to the algebraic solvers are as follows:

- *algebra_system*: function literal referring to a function specifying the system of algebraic equations with signature `(vector, ...):vector`. The arguments represent (1) unknowns, (2) additional parameter and/or data arguments, and the return value contains the value of the algebraic function, which goes to 0 when we plug in the solution to the algebraic system,
- *y_guess*: initial guess for the solution, type `vector`,
- *...*: variadic arguments.

The algebraic solvers admit control parameters. While Stan provides default values, the user should be prepared to adjust the control parameters. The following controls are available:

- *scaling_step*: for the Newton solver only, the scaled-step stopping tolerance, type `real`, data only. If a Newton step is smaller than the scaling step tolerance, the code breaks, assuming the solver is no longer making significant progress. If set to 0, this constraint is ignored. Default value is 10^{-3} .

- *rel_tol*: for the Powell solver only, the relative tolerance, type `real`, data only. The relative tolerance is the estimated relative error of the solver and serves to test if a satisfactory solution has been found. Default value is 10^{-10} .
- *function_tol*: function tolerance for the algebraic solver, type `real`, data only. After convergence of the solver, the proposed solution is plugged into the algebraic system and its norm is compared to the function tolerance. If the norm is below the function tolerance, the solution is deemed acceptable. Default value is 10^{-6} .
- *max_num_steps*: maximum number of steps to take in the algebraic solver, type `int`, data only. If the solver reaches this number of steps, it breaks and returns an error message. Default value is 200.

The difference in which control parameters are available has to do with the underlying implementations for the solvers and the control parameters these implementations support. The Newton solver is based on KINSOL from the SUNDIAL suites, while the Powell solver uses a module from the Eigen library.

Return value

The return value for the algebraic solver is an object of type `vector`, with values which, when plugged in as `y` make the algebraic function go to 0 (approximately, within the specified function tolerance).

Sizes and parallel arrays

Certain sizes have to be consistent. The initial guess, return value of the solver, and return value of the algebraic function must all be the same size.

Algorithmic details

Stan offers two methods to solve algebraic equations. `solve_newton` and `solve_newton_tol` use the Newton method, a first-order derivative based numerical solver. The Stan code builds on the implementation in KINSOL from the SUNDIALS suite (Hindmarsh et al. 2005). For many problems, we find that the Newton method is faster than the Powell method. If however Newton’s method performs poorly, either failing to or requiring an excessively long time to converge, the user should be prepared to switch to the Powell method.

`solve_powell` and `solve_powell_tol` are based on the Powell hybrid method (Powell 1970), which also uses first-order derivatives. The Stan code builds on the implementation of the hybrid solver in the unsupported module for nonlinear optimization problems of the Eigen library (Guennebaud, Jacob, et al. 2010). This solver is in turn based on the algorithm developed for the package MINPACK-1 (Jorge J. More 1980).

For both solvers, derivatives are propagated through the solution to the algebraic solution using the implicit function theorem and an adjoint method of automatic differentiation; for a discussion on this topic, see (Gaebler 2021) and (Margossian and Betancourt 2022).

11.2. Ordinary differential equation (ODE) solvers

Stan provides several higher order functions for solving initial value problems specified as Ordinary Differential Equations (ODEs).

Solving an initial value ODE means given a set of differential equations $y'(t, \theta) = f(t, y, \theta)$ and initial conditions $y(t_0, \theta)$, solving for y at a sequence of times $t_0 < t_1 < t_2, \dots < t_n$. $f(t, y, \theta)$ is referred to here as the ODE system function.

$f(t, y, \theta)$ will be defined as a function with a certain signature and provided along with the initial conditions and output times to one of the ODE solver functions.

To make it easier to write ODEs, the solve functions take extra arguments that are passed along unmodified to the user-supplied system function. Because there can be any number of these arguments and they can be of different types, they are denoted below as `...`. The types of the arguments represented by `...` in the ODE solve function call must match the types of the arguments represented by `...` in the user-supplied system function.

Non-stiff solver

```
array[] vector      ode_rk45(function ode, vector initial_state, real
initial_time, array[] real times, ...)
```

Solves the ODE system for the times provided using the Dormand-Prince algorithm, a 4th/5th order Runge-Kutta method.

Available since 2.24

```
array[] vector      ode_rk45_tol(function ode, vector initial_state,
real initial_time, array[] real times, data real rel_tol, data
real abs_tol, int max_num_steps, ...)
```

Solves the ODE system for the times provided using the Dormand-Prince algorithm, a 4th/5th order Runge-Kutta method with additional control parameters for the solver.

Available since 2.24

```
array[] vector      ode_ckrk(function ode, vector initial_state, real
initial_time, array[] real times, ...)
```

Solves the ODE system for the times provided using the Cash-Karp algorithm, a

4th/5th order explicit Runge-Kutta method.

Available since 2.27

```
array[] vector ode_ckrk_tol(function ode, vector initial_state,
real initial_time, array[] real times, data real rel_tol, data
real abs_tol, int max_num_steps, ...)
```

Solves the ODE system for the times provided using the Cash-Karp algorithm, a 4th/5th order explicit Runge-Kutta method with additional control parameters for the solver.

Available since 2.27

```
array[] vector ode_adams(function ode, vector initial_state, real
initial_time, array[] real times, ...)
```

Solves the ODE system for the times provided using the Adams-Moulton method.

Available since 2.24

```
array[] vector ode_adams_tol(function ode, vector initial_state,
real initial_time, array[] real times, data real rel_tol, data
real abs_tol, int max_num_steps, ...)
```

Solves the ODE system for the times provided using the Adams-Moulton method with additional control parameters for the solver.

Available since 2.24

Stiff solver

```
array[] vector ode_bdf(function ode, vector initial_state, real
initial_time, array[] real times, ...)
```

Solves the ODE system for the times provided using the backward differentiation formula (BDF) method.

Available since 2.24

```
array[] vector ode_bdf_tol(function ode, vector initial_state, real
initial_time, array[] real times, data real rel_tol, data real
abs_tol, int max_num_steps, ...)
```

Solves the ODE system for the times provided using the backward differentiation formula (BDF) method with additional control parameters for the solver.

Available since 2.24

Adjoint solver

```
array[] vector      ode_adjoint_tol_ctl(function ode, vector initial_state, real initial_time, array[] real times, data real rel_tol_forward, data vector abs_tol_forward, data real rel_tol_backward, data vector abs_tol_backward, int max_num_steps, int num_steps_between_checkpoints, int interpolation_polynomial, int solver_forward, int solver_backward, ...)
```

Solves the ODE system for the times provided using the adjoint ODE solver method from CVODES. The adjoint ODE solver requires a checkpointed forward in time ODE integration, a backwards in time integration that makes uses of an interpolated version of the forward solution, and the solution of a quadrature problem (the number of which depends on the number of parameters passed to the solve). The tolerances and numeric methods used for the forward solve, backward solve, quadratures, and interpolation can all be configured.

Available since 2.27

ODE system function

The first argument to one of the ODE solvers is always the ODE system function. The ODE system function must have a `vector` return type, and the first two arguments must be a `real` and `vector` in that order. These two arguments are followed by the variadic arguments that are passed through from the ODE solve function call:

```
vector ode(real time, vector state, ...)
```

The ODE system function should return the derivative of the state with respect to time at the time and state provided. The length of the returned vector must match the length of the state input into the function.

The arguments to this function are:

- `time`, the time to evaluate the ODE system
- `state`, the state of the ODE system at the time specified
- `...`, sequence of arguments passed unmodified from the ODE solve function call. The types here must match the types in the `...` arguments of the ODE solve function call.

Arguments to the ODE solvers

The arguments to the ODE solvers in both the stiff and non-stiff solvers are the same. The arguments to the adjoint ODE solver are different; see [Arguments to the](#)

adjoint ODE solver.

- *ode*: ODE system function,
- *initial_state*: initial state, type `vector`,
- *initial_time*: initial time, type `real`,
- *times*: solution times, type `array[] real`,
- *...*: sequence of arguments that will be passed through unmodified to the ODE system function. The types here must match the types in the *...* arguments of the ODE system function.

For the versions of the `ode` solver functions ending in `_tol`, these three parameters must be provided after `times` and before the *...* arguments:

- `data rel_tol`: relative tolerance for the ODE solver, type `real`, data only,
- `data abs_tol`: absolute tolerance for the ODE solver, type `real`, data only, and
- `max_num_steps`: maximum number of steps to take between output times in the ODE solver, type `int`, data only.

Because the tolerances are data arguments, they must be defined in either the data or transformed data blocks. They cannot be parameters, transformed parameters or functions of parameters or transformed parameters.

Arguments to the adjoint ODE solver

The arguments to the adjoint ODE solver are different from those for the other functions (for those see [Arguments to the ODE solvers](#)).

- *ode*: ODE system function,
- *initial_state*: initial state, type `vector`,
- *initial_time*: initial time, type `real`,
- *times*: solution times, type `array[] real`,
- `data rel_tol_forward`: Relative tolerance for forward solve, type `real`, data only,
- `data abs_tol_forward`: Absolute tolerance vector for each state for forward solve, type `vector`, data only,

- `data rel_tol_backward`: Relative tolerance for backward solve, type `real`, data only,
- `data abs_tol_backward`: Absolute tolerance vector for each state for backward solve, type `vector`, data only,
- `data rel_tol_quadrature`: Relative tolerance for backward quadrature, type `real`, data only,
- `data abs_tol_quadrature`: Absolute tolerance for backward quadrature, type `real`, data only,
- `data max_num_steps`: Maximum number of time-steps to take in integrating the ODE solution between output time points for forward and backward solve, type `int`, data only,
- `num_steps_between_checkpoints`: number of steps between checkpointing forward solution, type `int`, data only,
- `interpolation_polynomial`: can be 1 for hermite or 2 for polynomial interpolation method of CVODES, type `int`, data only,
- `solver_forward`: solver used for forward ODE problem: 1=Adams (non-stiff), 2=BDF (stiff), type `int`, data only,
- `solver_backward`: solver used for backward ODE problem: 1=Adams (non-stiff), 2=BDF (stiff), type `int`, data only.
- `...`: sequence of arguments that will be passed through unmodified to the ODE system function. The types here must match the types in the `...` arguments of the ODE system function.

Because the tolerances are data arguments, they must be defined in either the data or transformed data blocks. They cannot be parameters, transformed parameters or functions of parameters or transformed parameters.

Return values

The return value for the ODE solvers is an array of vectors (type `array[] vector`), one vector representing the state of the system at every time in specified in the `times` argument.

Array and vector sizes

The sizes must match, and in particular, the following groups are of the same size:

- state variables passed into the system function, derivatives returned by the system function, initial state passed into the solver, and length of each vector

in the output,

- number of solution times and number of vectors in the output.

11.3. Differential-Algebraic equation (DAE) solver

Stan provides two higher order functions for solving initial value problems specified as Differential-Algebraic Equations (DAEs) with index-1 (Serban et al. 2021).

Solving an initial value DAE means given a set of residual functions $r(y'(t, \theta), y(t, \theta), t)$ and initial conditions $(y(t_0, \theta), y'(t_0, \theta))$, solving for y at a sequence of times $t_0 < t_1 \leq t_2, \dots \leq t_n$. The residual function $r(y', y, t, \theta)$ will be defined as a function with a certain signature and provided along with the initial conditions and output times to one of the DAE solver functions.

Similar to ODE solvers, the DAE solver function takes extra arguments that are passed along unmodified to the user-supplied system function. Because there can be any number of these arguments and they can be of different types, they are denoted below as \dots , and the types of these arguments, also represented by \dots in the DAE solver call, must match the types of the arguments represented by \dots in the user-supplied system function.

The DAE solver

```
array[] vector    dae(function residual, vector initial_state, vector initial_state_derivative, data real initial_time, data array[] real times, ...)
```

Solves the DAE system using the backward differentiation formula (BDF) method (Serban et al. 2021).

Available since 2.29

```
array[] vector    dae_tol(function residual, vector initial_state, vector initial_state_derivative, data real initial_time, data array[] real times, data real rel_tol, data real abs_tol, int max_num_steps, ...)
```

Solves the DAE system for the times provided using the backward differentiation formula (BDF) method with additional control parameters for the solver.

Available since 2.29

DAE system function

The first argument to the DAE solver is the DAE residual function. The DAE residual function must have a `vector` return type, and the first three arguments must be a `real`, `vector`, and `vector`, in that order. These three arguments are

followed by the variadic arguments that are passed through from the DAE solver function call:

```
vector residual(real time, vector state, vector state_derivative, ...)
```

The DAE residual function should return the residuals at the time and state provided. The length of the returned vector must match the length of the state input into the function.

The arguments to this function are:

- *time*, the time to evaluate the DAE system
- *state*, the state of the DAE system at the time specified
- *state_derivative*, the time derivatives of the state of the DAE system at the time specified
- ..., sequence of arguments passed unmodified from the DAE solve function call. The types here must match the types in the ... arguments of the DAE solve function call.

Arguments to the DAE solver

The arguments to the DAE solver are

- *residual*: DAE residual function,
- *initial_state*: initial state, type `vector`,
- *initial_state_derivative*: time derivative of the initial state, type `vector`,
- *initial_time*: initial time, type `data real`,
- *times*: solution times, type `data array[] real`,
- ...: sequence of arguments that will be passed through unmodified to the DAE residual function. The types here must match the types in the ... arguments of the DAE residual function.

For `dae_tol`, the following three parameters must be provided after `times` and before the ... arguments:

- `data rel_tol`: relative tolerance for the DAE solver, type `real`, data only,
- `data abs_tol`: absolute tolerance for the DAE solver, type `real`, data only, and

- `max_num_steps`: maximum number of steps to take between output times in the DAE solver, type `int`, data only.

Because the tolerances are data arguments, they must be supplied as primitive numerics or defined in either the data or transformed data blocks. They cannot be parameters, transformed parameters or functions of parameters or transformed parameters.

Consistency of the initial conditions

The user is responsible to ensure the residual function becomes zero at the initial time, `t0`, when the arguments `initial_state` and `initial_state_derivative` are introduced as `state` and `state_derivative`, respectively.

Return values

The return value for the DAE solvers is an array of vectors (type `array[] vector`), one vector representing the state of the system at every time specified in the `times` argument.

Array and vector sizes

The sizes must match, and in particular, the following groups are of the same size:

- state variables and state derivatives passed into the residual function, the residual returned by the residual function, initial state and initial state derivatives passed into the solver, and length of each vector in the output,
- number of solution times and number of vectors in the output.

11.4. 1D integrator

Stan provides a built-in mechanism to perform 1D integration of a function via quadrature methods.

It operates similarly to the [algebraic solver](#) and the [ordinary differential equations solver](#) in that it allows as an argument a function.

Like both of those utilities, some of the arguments are limited to data only expressions. These expressions must not contain variables other than those declared in the data or transformed data blocks.

Specifying an integrand as a function

Performing a 1D integration requires the integrand to be specified somehow. This is done by defining a function in the Stan functions block with the special signature:

```
real integrand(real x, real xc, array[] real theta,
               array[] real x_r, array[] int x_i)
```

The function should return the value of the integrand evaluated at the point x .

The argument of this function are:

- x , the independent variable being integrated over
- xc , a high precision version of the distance from x to the nearest endpoint in a definite integral (for more into see section [Precision Loss](#)).
- θ , parameter values used to evaluate the integral
- x_r , data values used to evaluate the integral
- x_i , integer data used to evaluate the integral

Like algebraic solver and the differential equations solver, the 1D integrator separates parameter values, θ , from data values, x_r .

Call to the 1D integrator

```
real integrate_1d (function integrand, real a, real b, array[] real
theta, array[] real x_r, array[] int x_i)
```

Integrates the integrand from a to b .

Available since 2.23

```
real integrate_1d (function integrand, real a, real b, array[] real
theta, array[] real x_r, array[] int x_i, real relative_tolerance)
```

Integrates the integrand from a to b with the given relative tolerance.

Available since 2.23

Arguments to the 1D integrator

The arguments to the 1D integrator are as follows:

- *integrand*: function literal referring to a function specifying the integrand with signature (real, real, array[] real, array[] real, array[] int):real The arguments represent
 - (1) where integrand is evaluated,
 - (2) distance from evaluation point to integration limit for definite integrals,
 - (3) parameters,
 - (4) real data

- (5) integer data, and the return value is the integrand evaluated at the given point,
- a : left limit of integration, may be negative infinity, type `real`,
- b : right limit of integration, may be positive infinity, type `real`,
- θ : parameters only, type `array[] real`,
- x_r : real data only, type `array[] real`,
- x_i : integer data only, type `array[] int`.

A `relative_tolerance` argument can optionally be provided for more control over the algorithm:

- `relative_tolerance`: relative tolerance for the 1d integrator, type `real`, data only.

Return value

The return value for the 1D integrator is a `real`, the value of the integral.

Zero-crossing integrals

For numeric stability, integrals on the (possibly infinite) interval (a, b) that cross zero are split into two integrals, one from $(a, 0)$ and one from $(0, b)$. Each integral is separately integrated to the given `relative_tolerance`.

Precision loss near limits of integration in definite integrals

When integrating certain definite integrals, there can be significant precision loss in evaluating the integrand near the endpoints. This has to do with the breakdown in precision of double precision floating point values when adding or subtracting a small number from a number much larger than it in magnitude (for instance, $1.0 - x$). `xc` (as passed to the integrand) is a high-precision version of the distance between `x` and the definite integral endpoints and can be used to address this issue. More information (and an example where this is useful) is given in the User's Guide. For zero crossing integrals, `xc` will be a high precision version of the distance to the endpoints of the two smaller integrals. For any integral with an endpoint at negative infinity or positive infinity, `xc` is set to NaN.

Algorithmic details

Internally the 1D integrator uses the double-exponential methods in the Boost 1D quadrature library. Boost in turn makes use of quadrature methods developed in (Takahasi and Mori 1974), (Mori 1978), (Bailey, Jeyabalan, and Li 2005), and (Tanaka et al. 2009).

The gradients of the integral are computed in accordance with the Leibniz integral rule. Gradients of the integrand are computed internally with Stan's automatic

differentiation.

11.5. Reduce-sum function

Stan provides a higher-order reduce function for summation. A function which returns a scalar $g: U \rightarrow \text{real}$ is mapped to every element of a list of type `array[] U, { x1, x2, ... }` and all the results are accumulated,

$$g(x_1) + g(x_2) + \dots$$

For efficiency reasons the reduce function doesn't work with the element-wise evaluated function g itself, but instead works through evaluating partial sums, $f: \text{array[]} U \rightarrow \text{real}$, where:

$$\begin{aligned} f(\{ x_1 \}) &= g(x_1) \\ f(\{ x_1, x_2 \}) &= g(x_1) + g(x_2) \\ f(\{ x_1, x_2, \dots \}) &= g(x_1) + g(x_2) + \dots \end{aligned}$$

Mathematically the summation reduction is associative and forming arbitrary partial sums in an arbitrary order will not change the result. However, floating point numerics on computers only have a limited precision such that associativity does not hold exactly. This implies that the order of summation determines the exact numerical result. For this reason, the higher-order reduce function is available in two variants:

- `reduce_sum`: Automatically choose partial sums partitioning based on a dynamic scheduling algorithm.
- `reduce_sum_static`: Compute the same sum as `reduce_sum`, but partition the input in the same way for given data set (in `reduce_sum` this partitioning might change depending on computer load). This should result in stable numerical evaluations.

Specifying the reduce-sum function

The higher-order reduce function takes a partial sum function f , an array argument x (with one array element for each term in the sum), a recommended `grainsize`, and a set of shared arguments. This representation allows parallelization of the resultant sum.

```
real reduce_sum(F f, array[] T x, int grainsize, T1 s1, T2 s2, ...)
```

```
real reduce_sum_static(F f, array[] T x, int grainsize, T1 s1, T2 s2, ...)
```

Returns the equivalent of `f(x, 1, size(x), s1, s2, ...)`, but computes the result in parallel by breaking the array `x` into independent partial sums. `s1`, `s2`, ... are shared between all terms in the sum.

Available since 2.23

- `f`: function literal referring to a function specifying the partial sum operation. Refer to the [partial sum function](#).
- `x`: array of `T`, one for each term of the reduction, `T` can be any type,
- `grainsize`: For `reduce_sum`, `grainsize` is the recommended size of the partial sum (`grainsize = 1` means pick totally automatically). For `reduce_sum_static`, `grainsize` determines the maximum size of the partial sums, type `int`,
- `s1`: first (optional) shared argument, type `T1`, where `T1` can be any type
- `s2`: second (optional) shared argument, type `T2`, where `T2` can be any type,
- `...`: remainder of shared arguments, each of which can be any type.

The partial sum function

The partial sum function must have the following signature where the type `T`, and the types of all the shared arguments (`T1, T2, ...`) match those of the original `reduce_sum` (`reduce_sum_static`) call.

```
(array[] T x_subset, int start, int end, T1 s1, T2 s2, ...):real
```

The partial sum function returns the sum of the `start` to `end` terms (inclusive) of the overall calculations. The arguments to the partial sum function are:

- `x_subset`, the subset of `x` a given partial sum is responsible for computing, type `array[] T`, where `T` matches the type of `x` in `reduce_sum` (`reduce_sum_static`)
- `start`, the index of the first term of the partial sum, type `int`
- `end`, the index of the last term of the partial sum (inclusive), type `int`
- `s1`, first shared argument, type `T1`, matching type of `s1` in `reduce_sum` (`reduce_sum_static`)
- `s2`, second shared argument, type `T2`, matching type of `s2` in `reduce_sum` (`reduce_sum_static`)
- `...`, remainder of shared arguments, with types matching those in `reduce_sum` (`reduce_sum_static`)

11.6. Map-rect function

Stan provides a higher-order map function. This allows map-reduce functionality to be coded in Stan as described in the user's guide.

Specifying the mapped function

The function being mapped must have a signature identical to that of the function `f` in the following declaration.

```
vector f(vector phi, vector theta,
         data array[] real x_r, data array[] int x_i);
```

The map function returns the sequence of results for the particular shard being evaluated. The arguments to the mapped function are:

- `phi`, the sequence of parameters shared across shards
- `theta`, the sequence of parameters specific to this shard
- `x_r`, sequence of real-valued data
- `x_i`, sequence of integer data

All input for the mapped function must be packed into these sequences and all output from the mapped function must be packed into a single vector. The vector of output from each mapped function is concatenated into the final result.

Rectangular map

The rectangular map function operates on rectangular (not ragged) data structures, with parallel data structures for job-specific parameters, job-specific real data, and job-specific integer data.

```
vector map_rect(F f, vector phi, array[] vector theta, data array[,] real x_r, data array[,] int x_i)
```

Return the concatenation of the results of applying the function `f`, of type `(vector, vector, array[] real, array[] int):vector` elementwise, i.e., `f(phi, theta[n], x_r[n], x_i[n])` for each `n` in `1:N`, where `N` is the size of the parallel arrays of job-specific/local parameters `theta`, real data `x_r`, and integer data `x_i`. The shared/global parameters `phi` are passed to each invocation of `f`.

Available since 2.18

12. Variable Transformation Functions

Variable transformation functions provide implementations of the built-in constraining and unconstraining transforms defined in [Stan Reference Manual](#).

For each of the built-in variable transforms there are three functions named after the transform with differing suffixes. A `_unconstrain` function that maps from the constrained space back to free variables (the “transform”), A `_constrain` function that maps from free variables to constrained variables (the “inverse transform”), and a [`_jacobian` function](#), which computes the same value as the `_constrain` function while also incrementing the Jacobian accumulator with the log Jacobian determinant.

For this page, variables named `y` are unconstrained, while variables named `x` are in the constrained space. The unconstraining functions will reject if their input does not satisfy the declared constraint.

12.1. Transforms for scalars

These transformations take unconstrained values on the real number line and either constrain the, to a subset of the real line with a lower bound, upper bound, or both, or provide an affine map that does not constrain values but can help with shifting and scaling them so they are more standardized.

The functions are all overloaded to apply to containers elementwise. If the `y` argument is a container, the others must be either scalars or containers of exactly the same size.

Lower bounds

These functions perform the transform and inverse transform described in the [Lower Bounded Scalar section](#).

```
reals lower_bound_constrain(reals y, reals lb)
```

Takes a value `y` and lower bound `lb` and returns the corresponding value which is greater than `lb` (except for the possibility of rounding due to numeric precision issues, in which case it will be equal to the bound).

Available since 2.37

```
reals lower_bound_jacobian(reals y, reals lb)
```

Takes a value y and lower bound lb and returns the corresponding value which is greater than lb (except for the possibility of rounding due to numeric precision issues, in which case it will be equal to the bound).

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

```
reals lower_bound_unconstrain(reals x, reals lb)
```

Takes a value x which is greater than lb and returns the corresponding unconstrained value.

Available since 2.37

Upper bounds

These functions perform the transform and inverse transform described in the [Upper Bounded Scalar section](#).

```
reals upper_bound_constrain(reals y, reals ub)
```

Takes a value y and upper bound ub and returns the corresponding value which is less than ub (except for the possibility of rounding due to numeric precision issues, in which case it will be equal to the bound).

Available since 2.37

```
reals upper_bound_jacobian(reals x, reals ub)
```

Takes a value y and upper bound ub and returns the corresponding value which is less than ub (except for the possibility of rounding due to numeric precision issues, in which case it will be equal to the bound).

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

```
reals upper_bound_unconstrain(reals x, reals ub)
```

Takes a value x which is less than ub and returns the corresponding unconstrained value.

Available since 2.37

Upper and lower bounds

These functions perform the transform and inverse transform described in the [Lower and Upper Bounded Scalar section](#).

`reals lower_upper_bound_constraint(reals y, reals lb, reals ub)`

Takes a value `y`, lower bound `lb`, and upper bound `ub` and returns the corresponding value which is bounded between `lb` and `ub` (except for the possibility of rounding due to numeric precision issues, in which case it will be equal to the bound).

Available since 2.37

`reals lower_upper_bound_jacobian(reals y, reals lb, reals ub)`

Takes a value `y`, lower bound `lb`, and upper bound `ub` and returns the corresponding value which is bounded between `lb` and `ub` (except for the possibility of rounding due to numeric precision issues, in which case it will be equal to the bound).

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

`reals lower_upper_bound_unconstrain(reals x, reals lb, reals ub)`

Takes a value `x` which is bounded between `lb` and `ub` and returns the corresponding unconstrained value.

Available since 2.37

Affine transforms

These functions perform the transform and inverse transform described in the [Affinely Transformed Scalar section](#).

`reals offset_multiplier_constraint(reals y, reals offset, reals mult)`

Takes a value `y`, shift `offset`, and scale `mult` and returns a rescaled and shifted value.

Available since 2.37

`reals offset_multiplier_jacobian(reals y, reals offset, reals mult)`

Takes a value `y`, shift `offset`, and scale `mult` and returns a rescaled and shifted value.

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

`reals offset_multiplier_unconstrain(reals x, reals offset, reals mult)`

Takes a value `x`, shift `offset`, and scale `mult` and a value which has been un-scaled and un-shifted.

Available since 2.37

12.2. Transforms for constrained vectors

These functions constrain entire vectors holistically. Some transforms also change the length of the vector, as noted in the documentation.

Where `vectors` is used, this indicates that either a `vector` or a (possibly multidimensional) array of `vectors` may be provided. The array will be processed element by element.

Ordered vectors

These functions perform the transform and inverse transform described in the [Ordered Vector section](#).

`vectors ordered_constraint(vectors y)`

Takes a free vector `y` and returns a vector with elements in ascending order.

Available since 2.37

`vectors ordered_jacobian(vectors y)`

Takes a free vector `y` and returns a vector with elements in ascending order.

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

`vectors ordered_unconstraint(vectors x)`

Takes an ordered vector `x` and returns the corresponding free vector.

Available since 2.37

Positive order vectors

These functions perform the transform and inverse transform described in the [Positive Ordered Vector section](#).

`vectors positive_ordered_constraint(vectors y)`

Takes a free vector `y` and returns a vector with positive elements in ascending order.

Available since 2.37

`vectors positive_ordered_jacobian(vectors y)`

Takes a free vector `y` and returns a vector with positive elements in ascending order.

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

`vectors positive_ordered_unconstrain(vectors x)`

Takes an ordered vector x with positive entries and returns the corresponding free vector.

Available since 2.37

Simplexes

These functions perform the transform and inverse transform described in the [Unit Simplex section](#).

`vectors simplex_constrain(vectors y)`

Takes a free vector y and returns a simplex (a vector such that each element is between 0 and 1, and the sum of the elements is 1, up to rounding errors).

This returned vector will have one extra element compared to the input y .

Available since 2.37

`vectors simplex_jacobian(vectors y)`

Takes a free vector y and returns a simplex (a vector such that each element is between 0 and 1, and the sum of the elements is 1, up to rounding errors).

This returned vector will have one extra element compared to the input y .

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

`vectors simplex_unconstrain(vectors x)`

Takes a simplex x and returns the corresponding free vector.

This returned vector will have one fewer elements compared to the input x .

Available since 2.37

Sum-to-zero vectors

These functions perform the transform and inverse transform described in the [Zero Sum Vector section](#).

`vectors sum_to_zero_constrain(vectors y)`

Takes a free vector y and returns a vector such that the elements sum to 0.

This returned vector will have one extra element compared to the input y .

Available since 2.37

`vectors sum_to_zero_jacobian(vectors y)`

Takes a free vector y and returns a vector such that the elements sum to 0.

The returned vector will have one extra element compared to the input y .

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

`vectors sum_to_zero_unconstrain(vectors x)`

Takes a vector x with elements that sum to 0 and returns the corresponding free vector.

This returned vector will have one fewer elements compared to the input x .

Available since 2.37

Unit vectors

These functions perform the transform and inverse transform described in the [Unit Vector section](#).

`vectors unit_vectors_constrain(vectors y)`

Takes a free vector y and returns a vector with unit length, i.e., $\text{norm2}(\text{unit_vectors_constrain}(y)) == 1$ for any y that has a positive and finite norm itself (if y does not, the function rejects). Note that, in particular, this implies the function rejects if given a vector of all zeros.

Available since 2.37

`vectors unit_vectors_jacobian(vectors y)`

Takes a free vector y and returns a vector with unit length. This function rejects if given a vector of all zeros.

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

`vectors unit_vectors_unconstrain(vectors x)`

Takes a vector x of unit length and returns the corresponding free vector.

Available since 2.37

12.3. Transforms for constrained matrices

Similarly to the above, `vectors` means a vector or array thereof, and `matrices` means a matrix or array thereof.

Cholesky factors of correlation matrices

These functions perform the transform and inverse transform described in the [Cholesky Factors of Correlation Matrices](#) section.

`matrices cholesky_factor_corr_constrain(vectors y, int K)`

Takes a vector `y` and integer `K`, where `length(y) == choose(K, 2)`, and returns a `K` by `K` Cholesky factor of a correlation matrix. This matrix is a Cholesky factor of a covariance matrix (i.e., a lower triangular matrix with a strictly positive diagonal), but with the additional constraint that each row is of unit length.

Available since 2.37

Takes a vector `y` and integer `K`, where `length(y) == choose(K, 2)`, and returns a `K` by `K` Cholesky factor of a correlation matrix.

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment. `matrices cholesky_factor_corr_jacobian(vectors y, int K)`

Available since 2.37

`vectors cholesky_factor_corr_unconstrain(matrices x)`

Takes `x`, a $(K \times K)$ matrix which is the Cholesky factor of a correlation matrix (a lower triangular matrix with a strictly positive diagonal and each row having unit length), and returns the corresponding free vector of length $\$ \times K$.

Available since 2.37

Cholesky factors of covariance matrices

These functions perform the transform and inverse transform described in the [Cholesky Factors of Covariance Matrices](#) section.

`matrices cholesky_factor_cov_constrain(vectors y, int M, int N)`

Takes a free vector `y` and integers `M` and `N` and returns the `M` by `N` Cholesky factor of a covariance matrix. This matrix is a lower triangular matrix L , with a strictly positive diagonal, such that $L^T L$ is positive definite.

Note that `y` must have length `N + choose(N, 2) + (M - N) * N`, and `M` must be greater than or equal to `N`.

Available since 2.37

matrices `cholesky_factor_cov_jacobian`(vectors y , int M , int N)

Takes a free vector y and integers M and N and returns the M by N Cholesky factor of a covariance matrix. This matrix is a lower triangular matrix L , with a strictly positive diagonal, such that $L^T L$ is positive definite.

Note that y must have length $N + \text{choose}(N, 2) + (M - N) * N$, and M must be greater than or equal to N .

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

vectors `cholesky_factor_cov_unconstrain`(matrices x)

Takes a $M \times N$ matrix x which is a Cholesky factor of a covariance matrix (a matrix L such that $L^T L$ is positive definite) and returns the corresponding free vector of length $N + \binom{N}{2} + (M - N)N$.

Available since 2.37

Correlation matrices

These functions perform the transform and inverse transform described in the [Correlation Matrices section](#).

matrices `corr_matrix_constrain`(vectors y , int K)

Takes a vector y and integer K , where $\text{length}(y) == \text{choose}(K, 2)$, and returns a K by K correlation matrix (a positive definite matrix with a unit diagonal).

Available since 2.37

matrices `corr_matrix_jacobian`(vectors y , int K)

Takes a vector y and integer K , where $\text{length}(y) == \text{choose}(K, 2)$, and returns a K by K correlation matrix (a positive definite matrix with a unit diagonal).

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

vectors `corr_matrix_unconstrain`(matrices x)

Takes a $K \times K$ matrix x which is a correlation matrix (a positive definite matrix with a unit diagonal) and returns the corresponding free vector of size $\binom{K}{2}$.

Available since 2.37

Covariance matrices

These functions perform the transform and inverse transform described in the [Covariance Matrices section](#).

matrices cov_matrix_constrain(vectors y, int K)

Takes a vector y and integer K , where $\text{length}(y) == K + \text{choose}(K, 2)$, and returns a K by K covariance matrix (a positive definite matrix).

Available since 2.37

matrices cov_matrix_jacobian(vectors y, int K)

Takes a vector y and integer K , where $\text{length}(y) == K + \text{choose}(K, 2)$, and returns a K by K covariance matrix (a positive definite matrix).

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

vectors cov_matrix_unconstrain(matrices x)

Takes a $K \times K$ positive definite matrix x and returns the corresponding free vector of size $K + \binom{K}{2}$.

Available since 2.37

Column-stochastic matrices

These functions perform the transform and inverse transform described in the [Stochastic Matrix section](#) for column (left) stochastic matrices.

matrices stochastic_column_constrain(matrices y)

Takes a free matrix y of size $N \times M$ and returns a left stochastic matrix (a matrix where each column is a simplex) of size $N + 1 \times M$.

Available since 2.37

matrices stochastic_column_jacobian(matrices y)

Takes a free matrix y of size $N \times M$ and returns a left stochastic matrix (a matrix where each column is a simplex) of size $N + 1 \times M$.

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

matrices stochastic_column_unconstrain(matrices x)

Takes a left stochastic matrix x of size $N + 1 \times M$ and returns the corresponding

free matrix of size $N \times M$.

Available since 2.37

Row-stochastic matrices

These functions perform the transform and inverse transform described in the [Stochastic Matrix section](#) for row (right) stochastic matrices.

```
matrices stochastic_row_constraint(matrices y)
```

Takes a free matrix y of size $N \times M$ and returns a right stochastic matrix (a matrix where each row is a simplex) of size $N \times M + 1$.

Available since 2.37

```
matrices stochastic_row_jacobian(matrices y)
```

Takes a free matrix y of size $N \times M$ and returns a right stochastic matrix (a matrix where each row is a simplex) of size $N \times M + 1$.

This function also increments the Jacobian accumulator with the corresponding change of variables adjustment.

Available since 2.37

```
matrices stochastic_row_unconstraint(matrices x)
```

Takes a right stochastic matrix x of size $N \times M + 1$ and returns the corresponding free matrix of size $N \times M$.

Available since 2.37

Sum-to-zero matrices

The sum-to-zero matrix transforms map between unconstrained values and matrices whose rows and columns sum to zero; full definitions of the function and Jacobian can be found in the [sum-to-zero matrix section of the Reference Manual](#).

```
matrices sum_to_zero_constraint(matrices y)
```

The constraining function maps an unconstrained $N \times M$ matrix to an $(N + 1) \times (M + 1)$ matrix for which the rows and columns all sum to zero. This function covers the incrementation of the log Jacobian because the incrementation is zero.

This returned matrix will have one extra row and column compared to the input y .

Available since 2.37

```
matrices sum_to_zero_jacobian(matrices y)
```

The constraining function maps an unconstrained $N \times M$ matrix to an $(N + 1) \times$

$(M + 1)$ matrix for which the rows and columns all sum to zero. Because the log Jacobian incrementation is zero, this is identical to `sum_to_zero_constrain`.

This returned matrix will have one extra row and column compared to the input `y`.

Available since 2.37

`matrices sum_to_zero_unconstrain(matrices x)`

This function maps a matrix with rows that sum to zero and columns that sum to zero to an unconstrained matrix with one fewer row and one fewer column.

Available since 2.37

13. Deprecated Functions

This appendix lists currently deprecated functionality along with how to replace it.

Starting in Stan 2.29, deprecated functions with drop in replacements (such as the renaming of `get_lp` or `multiply_log`) will be removed 3 versions later e.g., functions deprecated in Stan 2.20 will be removed in Stan 2.23 and placed in [Removed Functions](#). The Stan compiler can [automatically update](#) these on the behalf of the user for the entire deprecation window and at least one version following the removal.

13.1. Integer division with `operator/`

Degrecated: Using `/` with two integer arguments is interpreted as integer floor division, such that

$$1/2 = 0$$

This is deprecated due to its confusion with real-valued division, where

$$1.0/2.0 = 0.5$$

Replacement: Use the integer division operator `operator%/%` instead.

13.2. `integrate_ode_rk45`, `integrate_ode_adams`, `integrate_ode_bdf` ODE Integrators

These ODE integrator functions have been replaced by those described in [Ordinary Differential Equation \(ODE\) Solvers](#).

Specifying an ordinary differential equation as a function

A system of ODEs is specified as an ordinary function in Stan within the `functions` block. The ODE system function must have this function signature:

```
array[] real ode(real time, array[] real state, array[] real theta,  
array[] real x_r, array[] int x_i);
```

The ODE system function should return the derivative of the state with respect to time at the time provided. The length of the returned real array must match the length of the state input into the function.

The arguments to this function are:

- *time*, the time to evaluate the ODE system
- *state*, the state of the ODE system at the time specified
- *theta*, parameter values used to evaluate the ODE system
- *x_r*, data values used to evaluate the ODE system
- *x_i*, integer data values used to evaluate the ODE system.

The ODE system function separates parameter values, *theta*, from data values, *x_r*, for efficiency in computing the gradients of the ODE.

Non-stiff solver

```
array[,] real    integrate_ode_rk45(function ode, array[] real initial_state, real initial_time, array[] real times, array[] real theta, array[] real x_r, array[] int x_i)
```

Solves the ODE system for the times provided using the Dormand-Prince algorithm, a 4th/5th order Runge-Kutta method.

Available since 2.10, deprecated in 2.24

```
array[,] real    integrate_ode_rk45(function ode, array[] real initial_state, real initial_time, array[] real times, array[] real theta, array[] real x_r, array[] int x_i, real rel_tol, real abs_tol, int max_num_steps)
```

Solves the ODE system for the times provided using the Dormand-Prince algorithm, a 4th/5th order Runge-Kutta method with additional control parameters for the solver.

Available since 2.10, deprecated in 2.24

```
array[,] real    integrate_ode(function ode, array[] real initial_state, real initial_time, array[] real times, array[] real theta, array[] real x_r, array[] int x_i)
```

Solves the ODE system for the times provided using the Dormand-Prince algorithm, a 4th/5th order Runge-Kutta method.

Available since 2.10, deprecated in 2.24

13.2. INTEGRATE_ODE_RK45, INTEGRATE_ODE_ADAMS, INTEGRATE_ODE_BDF

```
array[,] real integrate_ode_adams(function ode, array[] real initial_state, real initial_time, array[] real times, array[] real theta, data array[] real x_r, data array[] int x_i)  
Solves the ODE system for the times provided using the Adams-Moulton method.
```

Available since 2.23, deprecated in 2.24

```
array[,] real integrate_ode_adams(function ode, array[] real initial_state, real initial_time, array[] real times, array[] real theta, data array[] real x_r, data array[] int x_i, data real rel_tol, data real abs_tol, data int max_num_steps)  
Solves the ODE system for the times provided using the Adams-Moulton method with additional control parameters for the solver.
```

Available since 2.23, deprecated in 2.24

Stiff solver

```
array[,] real integrate_ode_bdf(function ode, array[] real initial_state, real initial_time, array[] real times, array[] real theta, data array[] real x_r, data array[] int x_i)  
Solves the ODE system for the times provided using the backward differentiation formula (BDF) method.
```

Available since 2.10, deprecated in 2.24

```
array[,] real integrate_ode_bdf(function ode, array[] real initial_state, real initial_time, array[] real times, array[] real theta, data array[] real x_r, data array[] int x_i, data real rel_tol, data real abs_tol, data int max_num_steps)  
Solves the ODE system for the times provided using the backward differentiation formula (BDF) method with additional control parameters for the solver.
```

Available since 2.10, deprecated in 2.24

Arguments to the ODE solvers

The arguments to the ODE solvers in both the stiff and non-stiff cases are as follows.

- *ode*: function literal referring to a function specifying the system of differential equations with signature:

```
(real, array[] real, array[] real, data array[] real, data array[] int):array[] real
```

The arguments represent (1) time, (2) system state, (3) parameters, (4) real data, and (5) integer data, and the return value contains the derivatives with respect to time of the state,

- *initial_state*: initial state, type `array[] real`,
- *initial_time*: initial time, type `int` or `real`,
- *times*: solution times, type `array[] real`,
- *theta*: parameters, type `array[] real`,
- data *x_r*: real data, type `array[] real`, data only, and
- data *x_i*: integer data, type `array[] int`, data only.

For more fine-grained control of the ODE solvers, these parameters can also be provided:

- data *rel_tol*: relative tolerance for the ODE solver, type `real`, data only,
- data *abs_tol*: absolute tolerance for the ODE solver, type `real`, data only, and
- data *max_num_steps*: maximum number of steps to take in the ODE solver, type `int`, data only.

Return values

The return value for the ODE solvers is an array of type `array[,] real`, with values consisting of solutions at the specified times.

Sizes and parallel arrays

The sizes must match, and in particular, the following groups are of the same size:

- state variables passed into the system function, derivatives returned by the system function, initial state passed into the solver, and rows of the return value of the solver,
- solution times and number of rows of the return value of the solver,
- parameters, real data and integer data passed to the solver will be passed to the system function

13.3. `algebra_solver`, `algebra_solver_newton` algebraic solvers

These algebraic solver functions have been replaced by those described in [Algebraic Equation Solvers..](#)

Specifying an algebraic equation as a function

An algebraic system is specified as an ordinary function in Stan within the function block. The algebraic system function must have this signature:

```
vector algebra_system(vector y, vector theta,
                      data array[] real x_r, array[] int x_i)
```

The algebraic system function should return the value of the algebraic function which goes to 0, when we plug in the solution to the algebraic system.

The argument of this function are:

- y , the unknowns we wish to solve for
- θ , parameter values used to evaluate the algebraic system
- x_r , data values used to evaluate the algebraic system
- x_i , integer data used to evaluate the algebraic system

The algebraic system function separates parameter values, θ , from data values, x_r , for efficiency in propagating the derivatives through the algebraic system.

Call to the algebraic solver

```
vector algebra_solver(function algebra_system, vector y_guess, vector theta, data array[] real x_r, array[] int x_i)
```

Solves the algebraic system, given an initial guess, using the Powell hybrid algorithm.

Available since 2.17, deprecated in 2.31

```
vector algebra_solver(function algebra_system, vector y_guess, vector theta, data array[] real x_r, array[] int x_i, data real rel_tol, data real f_tol, int max_steps)
```

Solves the algebraic system, given an initial guess, using the Powell hybrid algorithm with additional control parameters for the solver.

Available since 2.17, deprecated in 2.31

Note: In future releases, the function `algebra_solver` will be deprecated and replaced with `algebra_solver_powell`.

```
vector algebra_solver_newton(function algebra_system, vector y_guess, vector theta, data array[] real x_r, array[] int x_i)
```

Solves the algebraic system, given an initial guess, using Newton's method.

Available since 2.24, deprecated in 2.31

```
vector algebra_solver_newton(function algebra_system, vector y_guess, vector theta, data array[] real x_r, array[] int x_i,
```

```
data real rel_tol, data real f_tol, int max_steps)
```

Solves the algebraic system, given an initial guess, using Newton's method with additional control parameters for the solver.

Available since 2.24, deprecated in 2.31

Arguments to the algebraic solver

The arguments to the algebraic solvers are as follows:

- *algebra_system*: function literal referring to a function specifying the system of algebraic equations with signature (vector, vector, array[] real, array[] int):vector. The arguments represent (1) unknowns, (2) parameters, (3) real data, and (4) integer data, and the return value contains the value of the algebraic function, which goes to 0 when we plug in the solution to the algebraic system,
- *y_guess*: initial guess for the solution, type vector,
- *theta*: parameters only, type vector,
- *x_r*: real data only, type array[] real, and
- *x_i*: integer data only, type array[] int.

For more fine-grained control of the algebraic solver, these parameters can also be provided:

- *rel_tol*: relative tolerance for the algebraic solver, type real, data only,
- *function_tol*: function tolerance for the algebraic solver, type real, data only,
- *max_num_steps*: maximum number of steps to take in the algebraic solver, type int, data only.

Return value

The return value for the algebraic solver is an object of type vector, with values which, when plugged in as y make the algebraic function go to 0.

Sizes and parallel arrays

Certain sizes have to be consistent. The initial guess, return value of the solver, and return value of the algebraic function must all be the same size.

The parameters, real data, and integer data will be passed from the solver directly to the system function.

14. Removed Functions

Functions which once existed in the Stan language and have since been replaced or removed are listed here.

14.1. `multiply_log` and `binomial_coefficient_log` functions

Removed: Currently two non-conforming functions ending in suffix `_log`.

Replacement: Replace `multiply_log(...)` with `lmultiply(...)`. Replace `binomial_coefficient_log(...)` with `lchoose(...)`.

Removed In: Stan 2.33

14.2. `get_lp()` function

Removed: The built-in no-argument function `get_lp()` is deprecated.

Replacement: Use the no-argument function `target()` instead.

Removed In: Stan 2.33

14.3. `fabs` function

Removed: The unary function `fabs` is deprecated.

Replacement: Use the unary function `abs` instead. Note that the return type for `abs` is different for integer overloads, but this replacement is safe due to Stan's type promotion rules.

Removed In: Stan 2.33

14.4. Exponentiated quadratic covariance functions

These covariance functions have been replaced by those described in [Gaussian Process Covariance Functions](#)

With magnitude α and length scale l , the exponentiated quadratic kernel is:

$$k(x_i, x_j) = \alpha^2 \exp \left(-\frac{1}{2\rho^2} \sum_{d=1}^D (x_{i,d} - x_{j,d})^2 \right)$$

matrix cov_exp_quad(row_vectors x, real alpha, real rho)
 The covariance matrix with an exponentiated quadratic kernel of x.

Available since 2.16, deprecated since 2.20, removed in in 2.33

matrix cov_exp_quad(vectors x, real alpha, real rho)
 The covariance matrix with an exponentiated quadratic kernel of x.

Available since 2.16, deprecated since 2.20, removed in in 2.33

matrix cov_exp_quad(array[] real x, real alpha, real rho)
 The covariance matrix with an exponentiated quadratic kernel of x.

Available since 2.16, deprecated since 2.20, removed in in 2.33

matrix cov_exp_quad(row_vectors x1, row_vectors x2, real alpha, real rho)

The covariance matrix with an exponentiated quadratic kernel of x1 and x2.

Available since 2.18, deprecated since 2.20, removed in in 2.33

matrix cov_exp_quad(vectors x1, vectors x2, real alpha, real rho)
 The covariance matrix with an exponentiated quadratic kernel of x1 and x2.

Available since 2.18, deprecated since 2.20, removed in in 2.33

matrix cov_exp_quad(array[] real x1, array[] real x2, real alpha, real rho)

The covariance matrix with an exponentiated quadratic kernel of x1 and x2.

Available since 2.18, deprecated since 2.20, removed in in 2.33

14.5. Real arguments to logical operators **operator&&**, **operator||**, and **operator!**

Removed: A nonzero real number (even NaN) was interpreted as true and a zero was interpreted as false.

Replacement: Explicit x != 0 comparison is preferred instead.

Removed In: Stan 2.34

15. Conventions for Probability Functions

Functions associated with distributions are set up to follow the same naming conventions for both built-in distributions and for user-defined distributions.

15.1. Suffix marks type of function

The suffix is determined by the type of function according to the following table.

function	outcome	suffix
log probability mass function	discrete	_lpmf
log probability density function	continuous	_lpdf
log cumulative distribution function	any	_lcdf
log complementary cumulative distribution function	any	_lccdf
random number generator	any	_rng

For example, `normal_lpdf` is the log of the normal probability density function (pdf) and `bernoulli_lpmf` is the log of the bernoulli probability mass function (pmf). The log of the corresponding cumulative distribution functions (cdf) use the same suffix, `normal_lcdf` and `bernoulli_lcdf`.

15.2. Argument order and the vertical bar

Each probability function has a specific outcome value and a number of parameters. Following conditional probability notation, probability density and mass functions use a vertical bar to separate the outcome from the parameters of the distribution. For example, `normal_lpdf(y | mu, sigma)` returns the value of mathematical formula $\log \text{Normal}(y | \mu, \sigma)$. Cumulative distribution functions separate the outcome from the parameters in the same way (e.g., `normal_lcdf(y_low | mu, sigma)`)

15.3. Sampling notation

The notation

```
y ~ normal(mu, sigma);
```

provides the same (proportional) contribution to the model log density as the explicit target density increment,

```
target += normal_lpdf(y | mu, sigma);
```

In both cases, the effect is to add terms to the target log density. The only difference is that the example with the sampling (\sim) notation drops all additive constants in the log density; the constants are not necessary for any of Stan’s sampling, approximation, or optimization algorithms.

15.4. Finite inputs

All of the distribution functions are configured to throw exceptions (effectively rejecting iterations or optimization steps) when they are supplied with non-finite arguments. The two cases of non-finite arguments are the infinite values and not-a-number value—these are standard in floating-point arithmetic.

15.5. Boundary conditions

Many distributions are defined with support or constraints on parameters forming an open interval. For example, the normal density function accepts a scale parameter $\sigma > 0$. If $\sigma = 0$, the probability function will throw an exception.

This is true even for (complementary) cumulative distribution functions, which will throw exceptions when given input that is out of the support.

15.6. Pseudorandom number generators

For most of the probability functions, there is a matching pseudorandom number generator (PRNG) with the suffix `_rng`. For example, the function `normal_rng(real, real)` accepts two real arguments, an unconstrained location μ and positive scale $\sigma > 0$, and returns an unconstrained pseudorandom value drawn from $\text{Normal}(\mu, \sigma)$. There are also vectorized forms of random number generators which return more than one random variate at a time.

Restricted to transformed data and generated quantities

Unlike regular functions, the PRNG functions may only be used in the transformed data or generated quantities blocks.

Limited vectorization

Unlike the probability functions, only some of the PRNG functions are vectorized.

15.7. Cumulative distribution functions

For most of the univariate probability functions, there is a corresponding cumulative distribution function, log cumulative distribution function, and log complementary

cumulative distribution function.

For a univariate random variable Y with probability function $p_Y(y | \theta)$, the cumulative distribution function (CDF) F_Y is defined by

$$F_Y(y) = \Pr[Y \leq y] = \int_{-\infty}^y p(y | \theta) dy.$$

The complementary cumulative distribution function (CCDF) is defined as

$$\Pr[Y > y] = 1 - F_Y(y).$$

The reason to use CCDFs instead of CDFs in floating-point arithmetic is that it is possible to represent numbers very close to 0 (the closest you can get is roughly 10^{-300}), but not numbers very close to 1 (the closest you can get is roughly $1 - 10^{-15}$).

In Stan, there is a cumulative distribution function for each probability function. For instance, `normal_cdf(y | mu, sigma)` is defined by

$$\int_{-\infty}^y \text{Normal}(y | \mu, \sigma) dy.$$

There are also log forms of the CDF and CCDF for most univariate distributions. For example, `normal_lcdf(y | mu, sigma)` is defined by

$$\log \left(\int_{-\infty}^y \text{Normal}(y | \mu, \sigma) dy \right)$$

and `normal_lccdf(y | mu, sigma)` is defined by

$$\log \left(1 - \int_{-\infty}^y \text{Normal}(y | \mu, \sigma) dy \right).$$

15.8. Vectorization

Stan's univariate log probability functions, including the log density functions, log mass functions, log CDFs, and log CCDFs, all support vectorized function application, with results defined to be the sum of the elementwise application of the function. Some of the PRNG functions support vectorization, see section [vectorized PRNG functions](#) for more details.

In all cases, matrix operations are at least as fast and usually faster than loops and vectorized log probability functions are faster than their equivalent form defined

with loops. This isn't because loops are slow in Stan, but because more efficient automatic differentiation can be used. The efficiency comes from the fact that a vectorized log probability function only introduces one new node into the expression graph, thus reducing the number of virtual function calls required to compute gradients in C++, as well as from allowing caching of repeated computations.

Stan also overloads the multivariate normal distribution, including the Cholesky-factor form, allowing arrays of row vectors or vectors for the variate and location parameter. This is a huge savings in speed because the work required to solve the linear system for the covariance matrix is only done once.

Stan also overloads some scalar functions, such as `log` and `exp`, to apply to vectors (arrays) and return vectors (arrays). These vectorizations are defined elementwise and unlike the probability functions, provide only minimal efficiency speedups over repeated application and assignment in a loop.

Vectorized function signatures

Vectorized scalar arguments

The normal probability function is specified with the signature

```
normal_lpdf(reals | reals, reals);
```

The pseudotype `reals` is used to indicate that an argument position may be vectorized. Argument positions declared as `reals` may be filled with a real, a one-dimensional array, a vector, or a row-vector. If there is more than one array or vector argument, their types can be anything but their size must match. For instance, it is legal to use `normal_lpdf(row_vector | vector, real)` as long as the vector and row vector have the same size.

Vectorized vector and row vector arguments

The multivariate normal distribution accepting vector or array of vector arguments is written as

```
multi_normal_lpdf(vectors | vectors, matrix);
```

These arguments may be row vectors, column vectors, or arrays of row vectors or column vectors.

Vectorized integer arguments

The pseudotype `ints` is used for vectorized integer arguments. Where it appears either an integer or array of integers may be used.

Evaluating vectorized log probability functions

The result of a vectorized log probability function is equivalent to the sum of the evaluations on each element. Any non-vector argument, namely `real` or `int`, is repeated. For instance, if `y` is a vector of size `N`, `mu` is a vector of size `N`, and `sigma` is a scalar, then

```
ll = normal_lpdf(y | mu, sigma);
```

is just a more efficient way to write

```
ll = 0;
for (n in 1:N) {
    ll = ll + normal_lpdf(y[n] | mu[n], sigma);
}
```

With the same arguments, the vectorized sampling statement

```
y ~ normal(mu, sigma);
```

has the same effect on the total log probability as

```
for (n in 1:N) {
    y[n] ~ normal(mu[n], sigma);
}
```

Evaluating vectorized PRNG functions

Some PRNG functions accept sequences as well as scalars as arguments. Such functions are indicated by argument pseudotypes `reals` or `ints`. In cases of sequence arguments, the output will also be a sequence. For example, the following is allowed in the transformed data and generated quantities blocks.

```
vector[3] mu = // ...
array[3] real x = normal_rng(mu, 3);
```

Argument types

In the case of PRNG functions, arguments marked `ints` may be integers or integer arrays, whereas arguments marked `reals` may be integers or reals, integer or real arrays, vectors, or row vectors.

pseudotype allowable PRNG arguments

<code>ints</code>	<code>int, array[] int</code>
<code>reals</code>	<code>int, array[] int, real, array[] real, vector, row_vector</code>

Dimension matching

In general, if there are multiple non-scalar arguments, they must all have the same dimensions, but need not have the same type. For example, the `normal_rng` function may be called with one vector argument and one real array argument as long as they have the same number of elements.

```
vector[3] mu = // ...
array[3] real sigma = // ...
array[3] real x = normal_rng(mu, sigma);
```

Return type

The result of a vectorized PRNG function depends on the size of the arguments and the distribution's support. If all arguments are scalars, then the return type is a scalar. For a continuous distribution, if there are any non-scalar arguments, the return type is a real array (`array[] real`) matching the size of any of the non-scalar arguments, as all non-scalar arguments must have matching size. Discrete distributions return `ints` and continuous distributions return `reals`, each of appropriate size. The symbol `R` denotes such a return type.

Part II

Discrete Distributions

16. Binary Distributions

Binary probability distributions have support on $\{0, 1\}$, where 1 represents the value true and 0 the value false.

16.1. Bernoulli distribution

Probability mass function

If $\theta \in [0, 1]$, then for $y \in \{0, 1\}$,

$$\text{Bernoulli}(y | \theta) = \begin{cases} \theta & \text{if } y = 1, \text{ and} \\ 1 - \theta & \text{if } y = 0. \end{cases}$$

Distribution statement

`y ~ bernoulli(theta)`

Increment target log probability density with `bernoulli_lupmf(y | theta)`.

Available since 2.0

Stan Functions

`real bernoulli_lpmf(ints y | reals theta)`

The log Bernoulli probability mass of y given chance of success θ

Available since 2.12

`real bernoulli_lupmf(ints y | reals theta)`

The log Bernoulli probability mass of y given chance of success θ dropping constant additive terms

Available since 2.25

`real bernoulli_cdf(ints y | reals theta)`

The Bernoulli cumulative distribution function of y given chance of success θ

Available since 2.0

`real bernoulli_lcdf(ints y | reals theta)`

The log of the Bernoulli cumulative distribution function of y given chance of success θ

Available since 2.12

real bernoulli_lccdf(ints y | reals theta)

The log of the Bernoulli complementary cumulative distribution function of y given chance of success theta

Available since 2.12

ints bernoulli_rng(reals theta)

Generate a Bernoulli variate with chance of success theta or an array of Bernoulli variates given an array of thetas of the same dimensions; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

16.2. Bernoulli distribution, logit parameterization

Stan also supplies a direct parameterization in terms of a logit-transformed chance-of-success parameter. This parameterization is more numerically stable if the chance-of-success parameter is on the logit scale, as with the linear predictor in a logistic regression.

Probability mass function

If $\alpha \in \mathbb{R}$, then for $y \in \{0, 1\}$,

$$\text{BernoulliLogit}(y | \alpha) = \text{Bernoulli}(y | \text{logit}^{-1}(\alpha)) = \begin{cases} \text{logit}^{-1}(\alpha) & \text{if } y = 1, \text{ and} \\ 1 - \text{logit}^{-1}(\alpha) & \text{if } y = 0. \end{cases}$$

Distribution statement

$y \sim \text{bernoulli_logit}(\text{alpha})$

Increment target log probability density with `bernoulli_logit_lupmf(y | alpha)`.

Available since 2.0

Stan Functions

real bernoulli_logit_lpmf(ints y | reals alpha)

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha)`

Available since 2.12

real bernoulli_logit_lupmf(ints y | reals alpha)

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha)` dropping constant additive terms

Available since 2.25

R **bernoulli_logit_rng**(reals alpha)

Generate a Bernoulli variate with chance of success $\text{logit}^{-1}(\alpha)$; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

16.3. Bernoulli-logit generalized linear model (Logistic Regression)

Stan also supplies a single function for a generalized linear model with Bernoulli distribution and logit link function, i.e. a function for a logistic regression. This provides a more efficient implementation of logistic regression than a manually written regression in terms of a Bernoulli distribution and matrix multiplication.

Probability mass function

If $x \in \mathbb{R}^{n \cdot m}$, $\alpha \in \mathbb{R}^n$, $\beta \in \mathbb{R}^m$, then for $y \in \{0, 1\}^n$,

$$\begin{aligned} \text{BernoulliLogitGLM}(y \mid x, \alpha, \beta) &= \prod_{1 \leq i \leq n} \text{Bernoulli}(y_i \mid \text{logit}^{-1}(\alpha_i + x_i \cdot \beta)) \\ &= \prod_{1 \leq i \leq n} \begin{cases} \text{logit}^{-1}(\alpha_i + \sum_{1 \leq j \leq m} x_{ij} \cdot \beta_j) & \text{if } y_i = 1, \text{ and} \\ 1 - \text{logit}^{-1}(\alpha_i + \sum_{1 \leq j \leq m} x_{ij} \cdot \beta_j) & \text{if } y_i = 0. \end{cases} \end{aligned}$$

Distribution statement

y ~ **bernoulli_logit_glm**(x, alpha, beta)

Increment target log probability density with `bernoulli_logit_glm_lupmf(y | x, alpha, beta)`.

Available since 2.25

Stan Functions

real **bernoulli_logit_glm_lpmf**(int y | matrix x, real alpha, vector beta)

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha + x * beta)`.

Available since 2.23

real **bernoulli_logit_glm_lupmf**(int y | matrix x, real alpha, vector beta)

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha + x * beta)` dropping constant additive terms.

Available since 2.25

```
real bernoulli_logit_glm_lpmf(int y | matrix x, vector alpha, vector beta)
```

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha + x * beta)`.

Available since 2.23

```
real bernoulli_logit_glm_lupmf(int y | matrix x, vector alpha, vector beta)
```

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha + x * beta)` dropping constant additive terms.

Available since 2.25

```
real bernoulli_logit_glm_lpmf(array[] int y | row_vector x, real alpha, vector beta)
```

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha + x * beta)`.

Available since 2.23

```
real bernoulli_logit_glm_lupmf(array[] int y | row_vector x, real alpha, vector beta)
```

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha + x * beta)` dropping constant additive terms.

Available since 2.25

```
real bernoulli_logit_glm_lpmf(array[] int y | row_vector x, vector alpha, vector beta)
```

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha + x * beta)`.

Available since 2.23

```
real bernoulli_logit_glm_lupmf(array[] int y | row_vector x, vector alpha, vector beta)
```

The log Bernoulli probability mass of y given chance of success `inv_logit(alpha + x * beta)` dropping constant additive terms.

Available since 2.25

```
real bernoulli_logit_glm_lpmf(array[] int y | matrix x, real alpha,
vector beta)
```

The log Bernoulli probability mass of y given chance of success $\text{inv_logit}(\alpha + x * \beta)$.

Available since 2.18

```
real bernoulli_logit_glm_lupmf(array[] int y | matrix x, real al-
pha, vector beta)
```

The log Bernoulli probability mass of y given chance of success $\text{inv_logit}(\alpha + x * \beta)$ dropping constant additive terms.

Available since 2.25

```
real bernoulli_logit_glm_lpmf(array[] int y | matrix x, vector al-
pha, vector beta)
```

The log Bernoulli probability mass of y given chance of success $\text{inv_logit}(\alpha + x * \beta)$.

Available since 2.18

```
real bernoulli_logit_glm_lupmf(array[] int y | matrix x, vector al-
pha, vector beta)
```

The log Bernoulli probability mass of y given chance of success $\text{inv_logit}(\alpha + x * \beta)$ dropping constant additive terms.

Available since 2.25

```
array[] int bernoulli_logit_glm_rng(matrix x, vector alpha, vector
beta)
```

Generate an array of Bernoulli variates with chances of success $\text{inv_logit}(\alpha + x * \beta)$; may only be used in transformed data and generated quantities blocks.

Available since 2.29

```
array[] int bernoulli_logit_glm_rng(row_vector x, vector alpha,
vector beta)
```

Generate an array of Bernoulli variates with chances of success $\text{inv_logit}(\alpha + x * \beta)$; may only be used in transformed data and generated quantities blocks.

Available since 2.29

17. Bounded Discrete Distributions

Bounded discrete probability functions have support on $\{0, \dots, N\}$ for some upper bound N .

17.1. Binomial distribution

Probability mass function

Suppose $N \in \mathbb{N}$ and $\theta \in [0, 1]$, and $n \in \{0, \dots, N\}$.

$$\text{Binomial}(n \mid N, \theta) = \binom{N}{n} \theta^n (1 - \theta)^{N - n}.$$

Log probability mass function

$$\begin{aligned} \log \text{Binomial}(n \mid N, \theta) &= \log \Gamma(N + 1) - \log \Gamma(n + 1) - \log \Gamma(N - n + 1) \\ &\quad + n \log \theta + (N - n) \log(1 - \theta), \end{aligned}$$

Gradient of log probability mass function

$$\frac{\partial}{\partial \theta} \log \text{Binomial}(n \mid N, \theta) = \frac{n}{\theta} - \frac{N - n}{1 - \theta}$$

Distribution statement

`n ~ binomial(N, theta)`

Increment target log probability density with `binomial_lupmf(n | N, theta)`.

Available since 2.0

Stan functions

`real binomial_lpmf(ints n | ints N, reals theta)`

The log binomial probability mass of n successes in N trials given chance of success θ

Available since 2.12

real `binomial_lupmf`(ints n | ints N, reals theta)

The log binomial probability mass of n successes in N trials given chance of success theta dropping constant additive terms

Available since 2.25

real `binomial_cdf`(ints n | ints N, reals theta)

The binomial cumulative distribution function of n successes in N trials given chance of success theta

Available since 2.0

real `binomial_lcdf`(ints n | ints N, reals theta)

The log of the binomial cumulative distribution function of n successes in N trials given chance of success theta

Available since 2.12

real `binomial_lccdf`(ints n | ints N, reals theta)

The log of the binomial complementary cumulative distribution function of n successes in N trials given chance of success theta

Available since 2.12

R `binomial_rng`(ints N, reals theta)

Generate a binomial variate with N trials and chance of success theta; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

17.2. Binomial distribution, logit parameterization

Stan also provides a version of the binomial probability mass function distribution with the chance of success parameterized on the unconstrained logistic scale.

Probability mass function

Suppose $N \in \mathbb{N}$, $\alpha \in \mathbb{R}$, and $n \in \{0, \dots, N\}$. Then

$$\begin{aligned} \text{BinomialLogit}(n | N, \alpha) &= \text{Binomial}(n | N, \text{logit}^{-1}(\alpha)) \\ &= \binom{N}{n} \left(\text{logit}^{-1}(\alpha)\right)^n \left(1 - \text{logit}^{-1}(\alpha)\right)^{N-n}. \end{aligned}$$

Log probability mass function

$$\begin{aligned}\log \text{BinomialLogit}(n \mid N, \alpha) &= \log \Gamma(N+1) - \log \Gamma(n+1) - \log \Gamma(N-n+1) \\ &\quad + n \log \text{logit}^{-1}(\alpha) + (N-n) \log \left(1 - \text{logit}^{-1}(\alpha)\right),\end{aligned}$$

Gradient of log probability mass function

$$\frac{\partial}{\partial \alpha} \log \text{BinomialLogit}(n \mid N, \alpha) = \frac{n}{\text{logit}^{-1}(-\alpha)} - \frac{N-n}{\text{logit}^{-1}(\alpha)}$$

Distribution statement

`n ~ binomial_logit(N, alpha)`

Increment target log probability density with `binomial_logit_lupmf(n | N, alpha)`.

Available since 2.0

Stan functions

`real binomial_logit_lpmf(ints n | ints N, reals alpha)`

The log binomial probability mass of `n` successes in `N` trials given logit-scaled chance of success `alpha`

Available since 2.12

`real binomial_logit_lupmf(ints n | ints N, reals alpha)`

The log binomial probability mass of `n` successes in `N` trials given logit-scaled chance of success `alpha` dropping constant additive terms

Available since 2.25

17.3. Binomial-logit generalized linear model (Logistic Regression)

Stan also supplies a single function for a generalized linear model with binomial distribution and logit link function, i.e., a function for logistic regression with aggregated outcomes. This provides a more efficient implementation of logistic regression than a manually written regression in terms of a binomial distribution and matrix multiplication.

Probability mass function

Suppose $N \in \mathbb{N}$, $x \in \mathbb{R}^{n \cdot m}$, $\alpha \in \mathbb{R}^n$, $\beta \in \mathbb{R}^m$, and $n \in \{0, \dots, N\}$. Then

$$\text{BinomialLogitGLM}(n \mid N, x, \alpha, \beta) = \text{Binomial}(n \mid N, \text{logit}^{-1}(\alpha_i + x_i \cdot \beta)) \\ = \binom{N}{n} \left(\text{logit}^{-1}(\alpha_i + \sum_{1 \leq j \leq m} x_{ij} \cdot \beta_j) \right)^n \left(1 - \text{logit}^{-1}(\alpha_i + \sum_{1 \leq j \leq m} x_{ij} \cdot \beta_j) \right)^{N-n}.$$

Distribution statement

`n ~ binomial_logit_glm(N, x, alpha, beta)`

Increment target log probability density with `binomial_logit_glm_lupmf(n | N, x, alpha, beta)`.

Available since 2.34

Stan Functions

`real binomial_logit_glm_lpmf(int n | int N, matrix x, real alpha, vector beta)`

The log binomial probability mass of n given N trials and chance of success $\text{inv_logit}(\alpha + x \cdot \beta)$.

Available since 2.34

`real binomial_logit_glm_lupmf(int n | int N, matrix x, real alpha, vector beta)`

The log binomial probability mass of n given N trials and chance of success $\text{inv_logit}(\alpha + x \cdot \beta)$ dropping constant additive terms.

Available since 2.34

`real binomial_logit_glm_lpmf(int n | int N, matrix x, vector alpha, vector beta)`

The log binomial probability mass of n given N trials and chance of success $\text{inv_logit}(\alpha + x \cdot \beta)$.

Available since 2.34

`real binomial_logit_glm_lupmf(int n | int N, matrix x, vector alpha, vector beta)`

The log binomial probability mass of n given N trials and chance of success $\text{inv_logit}(\alpha + x \cdot \beta)$ dropping constant additive terms.

Available since 2.34

```
real      binomial_logit_glm_lpmf(array[] int n | array[] int N,
row_vector x, real alpha, vector beta)
```

The log binomial probability mass of n given N trials and chance of success
 $\text{inv_logit}(\alpha + x * \beta)$.

Available since 2.34

```
real      binomial_logit_glm_lupmf(array[] int n | array[] int N,
row_vector x, real alpha, vector beta)
```

The log binomial probability mass of n given N trials and chance of success
 $\text{inv_logit}(\alpha + x * \beta)$ dropping constant additive terms.

Available since 2.34

```
real      binomial_logit_glm_lpmf(array[] int n | array[] int N,
row_vector x, vector alpha, vector beta)
```

The log binomial probability mass of n given N trials and chance of success
 $\text{inv_logit}(\alpha + x * \beta)$.

Available since 2.34

```
real      binomial_logit_glm_lupmf(array[] int n | array[] int N,
row_vector x, vector alpha, vector beta)
```

The log binomial probability mass of n given N trials and chance of success
 $\text{inv_logit}(\alpha + x * \beta)$ dropping constant additive terms.

Available since 2.34

```
real  binomial_logit_glm_lpmf(array[] int n | array[] int N, matrix
x, real alpha, vector beta)
```

The log binomial probability mass of n given N trials and chance of success
 $\text{inv_logit}(\alpha + x * \beta)$.

Available since 2.34

```
real binomial_logit_glm_lupmf(array[] int n | array[] int N, matrix
x, real alpha, vector beta)
```

The log binomial probability mass of n given N trials and chance of success
 $\text{inv_logit}(\alpha + x * \beta)$ dropping constant additive terms.

Available since 2.34

```
real  binomial_logit_glm_lpmf(array[] int n | array[] int N, matrix
x, vector alpha, vector beta)
```

The log binomial probability mass of n given N trials and chance of success $\text{inv_logit}(\text{alpha} + \mathbf{x} * \text{beta})$.

Available since 2.34

```
real binomial_logit_glm_lupmf(array[] int n | array[] int N, matrix x, vector alpha, vector beta)
```

The log binomial probability mass of n given N trials and chance of success $\text{inv_logit}(\text{alpha} + \mathbf{x} * \text{beta})$ dropping constant additive terms.

Available since 2.34

17.4. Beta-binomial distribution

Probability mass function

If $N \in \mathbb{N}$, $\alpha \in \mathbb{R}^+$, and $\beta \in \mathbb{R}^+$, then for $n \in 0, \dots, N$,

$$\text{BetaBinomial}(n | N, \alpha, \beta) = \binom{N}{n} \frac{B(n + \alpha, N - n + \beta)}{B(\alpha, \beta)},$$

where the beta function $B(u, v)$ is defined for $u \in \mathbb{R}^+$ and $v \in \mathbb{R}^+$ by

$$B(u, v) = \frac{\Gamma(u) \Gamma(v)}{\Gamma(u + v)}.$$

Distribution statement

```
n ~ beta_binomial(N, alpha, beta)
```

Increment target log probability density with `beta_binomial_lupmf(n | N, alpha, beta)`.

Available since 2.0

Stan functions

```
real beta_binomial_lpmf(ints n | ints N, reals alpha, reals beta)
```

The log beta-binomial probability mass of n successes in N trials given prior success count (plus one) of α and prior failure count (plus one) of β

Available since 2.12

```
real beta_binomial_lupmf(ints n | ints N, reals alpha, reals beta)
```

The log beta-binomial probability mass of n successes in N trials given prior success count (plus one) of α and prior failure count (plus one) of β dropping constant additive terms

Available since 2.25

real beta_binomial_cdf(ints n | ints N, reals alpha, reals beta)

The beta-binomial cumulative distribution function of n successes in N trials given prior success count (plus one) of alpha and prior failure count (plus one) of beta

Available since 2.0

real beta_binomial_lcdf(ints n | ints N, reals alpha, reals beta)

The log of the beta-binomial cumulative distribution function of n successes in N trials given prior success count (plus one) of alpha and prior failure count (plus one) of beta

Available since 2.12

real beta_binomial_lccdf(ints n | ints N, reals alpha, reals beta)

The log of the beta-binomial complementary cumulative distribution function of n successes in N trials given prior success count (plus one) of alpha and prior failure count (plus one) of beta

Available since 2.12

R beta_binomial_rng(ints N, reals alpha, reals beta)

Generate a beta-binomial variate with N trials, prior success count (plus one) of alpha, and prior failure count (plus one) of beta; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

17.5. Hypergeometric distribution

Probability mass function

If $a \in \mathbb{N}$, $b \in \mathbb{N}$, and $N \in \{0, \dots, a+b\}$, then for $n \in \{\max(0, N-b), \dots, \min(a, N)\}$,

$$\text{Hypergeometric}(n | N, a, b) = \frac{\binom{a}{n} \binom{b}{N-n}}{\binom{a+b}{N}}.$$

Distribution statement

$n \sim \text{hypergeometric}(N, a, b)$

Increment target log probability density with `hypergeometric_lupmf(n | N, a, b)`.

Available since 2.0

Stan functions

real `hypergeometric_lpmf`(int n | int N, int a, int b)

The log hypergeometric probability mass of n successes in N trials given total success count of a and total failure count of b

Available since 2.12

real `hypergeometric_lupmf`(int n | int N, int a, int b)

The log hypergeometric probability mass of n successes in N trials given total success count of a and total failure count of b dropping constant additive terms

Available since 2.25

int `hypergeometric_rng`(int N, int a, int b)

Generate a hypergeometric variate with N trials, total success count of a, and total failure count of b; may only be used in transformed data and generated quantities blocks

Available since 2.18

17.6. Categorical distribution

Probability mass functions

If $N \in \mathbb{N}$, $N > 0$, and if $\theta \in \mathbb{R}^N$ forms an N -simplex (i.e., has nonnegative entries summing to one), then for $y \in \{1, \dots, N\}$,

$$\text{Categorical}(y | \theta) = \theta_y.$$

In addition, Stan provides a log-odds scaled categorical distribution,

$$\text{CategoricalLogit}(y | \beta) = \text{Categorical}(y | \text{softmax}(\beta)).$$

See [the definition of softmax](#) for the definition of the softmax function.

Distribution statement

`y ~ categorical(theta)`

Increment target log probability density with `categorical_lupmf(y | theta)` dropping constant additive terms.

Available since 2.0

Distribution statement

`y ~ categorical_logit(beta)`

Increment target log probability density with `categorical_logit_lupmf(y | beta)`.

Available since 2.4

Stan functions

All of the categorical distributions are vectorized so that the outcome `y` can be a single integer (type `int`) or an array of integers (type `array[] int`).

`real categorical_lpmf(ints y | vector theta)`

The log categorical probability mass function with outcome(s) `y` in $1 : N$ given N -vector of outcome probabilities `theta`. The parameter `theta` must have non-negative entries that sum to one, but it need not be a variable declared as a simplex.

Available since 2.12

`real categorical_lupmf(ints y | vector theta)`

The log categorical probability mass function with outcome(s) `y` in $1 : N$ given N -vector of outcome probabilities `theta` dropping constant additive terms. The parameter `theta` must have non-negative entries that sum to one, but it need not be a variable declared as a simplex.

Available since 2.25

`real categorical_logit_lpmf(ints y | vector beta)`

The log categorical probability mass function with outcome(s) `y` in $1 : N$ given log-odds of outcomes `beta`.

Available since 2.12

`real categorical_logit_lupmf(ints y | vector beta)`

The log categorical probability mass function with outcome(s) `y` in $1 : N$ given log-odds of outcomes `beta` dropping constant additive terms.

Available since 2.25

`int categorical_rng(vector theta)`

Generate a categorical variate with N -simplex distribution parameter `theta`; may only be used in transformed data and generated quantities blocks

Available since 2.0

```
int categorical_logit_rng(vector beta)
```

Generate a categorical variate with outcome in range $1 : N$ from log-odds vector beta; may only be used in transformed data and generated quantities blocks

Available since 2.16

17.7. Categorical logit generalized linear model (softmax regression)

Stan also supplies a single function for a generalized linear model with categorical distribution and logit link function, i.e. a function for a softmax regression. This provides a more efficient implementation of softmax regression than a manually written regression in terms of a categorical distribution and matrix multiplication.

Note that the implementation does not put any restrictions on the coefficient matrix β . It is up to the user to use a reference category, a suitable prior or some other means of identifiability. See Multi-logit in the [Stan User's Guide](#).

Probability mass functions

If $N, M, K \in \mathbb{N}$, $N, M, K > 0$, and if $x \in \mathbb{R}^{M \times K}$, $\alpha \in \mathbb{R}^N$, $\beta \in \mathbb{R}^{K \cdot N}$, then for $y \in \{1, \dots, N\}^M$,

$$\begin{aligned} \text{CategoricalLogitGLM}(y \mid x, \alpha, \beta) &= \prod_{1 \leq i \leq M} \text{CategoricalLogit}(y_i \mid \alpha + x_i \cdot \beta) \\ &= \prod_{1 \leq i \leq M} \text{Categorical}(y_i \mid \text{softmax}(\alpha + x_i \cdot \beta)). \end{aligned}$$

See [the definition of softmax](#) for the definition of the softmax function.

Distribution statement

```
y ~ categorical_logit_glm(x, alpha, beta)
```

Increment target log probability density with `categorical_logit_glm_lupmf(y | x, alpha, beta)`.

Available since 2.23

Stan functions

```
real categorical_logit_glm_lpmf(int y | row_vector x, vector alpha, matrix beta)
```

The log categorical probability mass function with outcome y in $1 : N$ given N -vector of log-odds of outcomes $\alpha + x * \beta$.

Available since 2.23

```
real categorical_logit_glm_lupmf(int y | row_vector x, vector alpha, matrix beta)
```

The log categorical probability mass function with outcome y in $1 : N$ given N -vector of log-odds of outcomes $\text{alpha} + x * \text{beta}$ dropping constant additive terms.

Available since 2.25

```
real categorical_logit_glm_lpmf(int y | matrix x, vector alpha, matrix beta)
```

The log categorical probability mass function with outcomes y in $1 : N$ given N -vector of log-odds of outcomes $\text{alpha} + x * \text{beta}$.

Available since 2.23

```
real categorical_logit_glm_lupmf(int y | matrix x, vector alpha, matrix beta)
```

The log categorical probability mass function with outcomes y in $1 : N$ given N -vector of log-odds of outcomes $\text{alpha} + x * \text{beta}$ dropping constant additive terms.

Available since 2.25

```
real categorical_logit_glm_lpmf(array[] int y | row_vector x, vector alpha, matrix beta)
```

The log categorical probability mass function with outcomes y in $1 : N$ given N -vector of log-odds of outcomes $\text{alpha} + x * \text{beta}$.

Available since 2.23

```
real categorical_logit_glm_lupmf(array[] int y | row_vector x, vector alpha, matrix beta)
```

The log categorical probability mass function with outcomes y in $1 : N$ given N -vector of log-odds of outcomes $\text{alpha} + x * \text{beta}$ dropping constant additive terms.

Available since 2.25

```
real categorical_logit_glm_lpmf(array[] int y | matrix x, vector alpha, matrix beta)
```

The log categorical probability mass function with outcomes y in $1 : N$ given N -vector of log-odds of outcomes $\text{alpha} + x * \text{beta}$.

Available since 2.23

```
real categorical_logit_glm_lupmf(array[] int y | matrix x, vector
alpha, matrix beta)
```

The log categorical probability mass function with outcomes y in $1 : N$ given N -vector of log-odds of outcomes $\text{alpha} + x * \text{beta}$ dropping constant additive terms.

Available since 2.25

17.8. Discrete range distribution

Probability mass functions

If $l, u \in \mathbb{Z}$ are lower and upper bounds ($l \leq u$), then for any integer $y \in \{l, \dots, u\}$,

$$\text{DiscreteRange}(y | l, u) = \frac{1}{u - l + 1}.$$

Distribution statement

```
y ~ discrete_range(l, u)
```

Increment the target log probability density with `discrete_range_lupmf(y | l, u)` dropping constant additive terms.

Available since 2.26

Stan functions

All of the discrete range distributions are vectorized so that the outcome y and the bounds l, u can be a single integer (type `int`) or an array of integers (type `array[] int`).

```
real discrete_range_lpmf(ints y | ints l, ints u)
```

The log probability mass function with outcome(s) y in $l : u$.

Available since 2.26

```
real discrete_range_lupmf(ints y | ints l, ints u)
```

The log probability mass function with outcome(s) y in $l : u$ dropping constant additive terms.

Available since 2.26

```
real discrete_range_cdf(ints y | ints l, ints u)
```

The discrete range cumulative distribution function for the given y , lower and upper bounds.

Available since 2.26

real `discrete_range_lcdf`(ints y | ints l, ints u)

The log of the discrete range cumulative distribution function for the given y, lower and upper bounds.

Available since 2.26

real `discrete_range_lccdf`(ints y | ints l, ints u)

The log of the discrete range complementary cumulative distribution function for the given y, lower and upper bounds.

Available since 2.26

ints `discrete_range_rng`(ints l, ints u)

Generate a discrete variate between the given lower and upper bounds; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.26

17.9. Ordered logistic distribution

Probability mass function

If $K \in \mathbb{N}$ with $K > 2$, $c \in \mathbb{R}^{K-1}$ such that $c_k < c_{k+1}$ for $k \in \{1, \dots, K-2\}$, and $\eta \in \mathbb{R}$, then for $k \in \{1, \dots, K\}$,

$$\text{OrderedLogistic}(k | \eta, c) = \begin{cases} 1 - \text{logit}^{-1}(\eta - c_1) & \text{if } k = 1, \\ \text{logit}^{-1}(\eta - c_{k-1}) - \text{logit}^{-1}(\eta - c_k) & \text{if } 1 < k < K, \text{ and} \\ \text{logit}^{-1}(\eta - c_{K-1}) - 0 & \text{if } k = K. \end{cases}$$

The $k = K$ case is written with the redundant subtraction of zero to illustrate the parallelism of the cases; the $k = 1$ and $k = K$ edge cases can be subsumed into the general definition by setting $c_0 = -\infty$ and $c_K = +\infty$ with $\text{logit}^{-1}(-\infty) = 0$ and $\text{logit}^{-1}(\infty) = 1$.

Distribution statement

k ~ `ordered_logistic`(eta, c)

Increment target log probability density with `ordered_logistic_lupmf(k | eta, c)`.

Available since 2.0

Stan functions

real ordered_logistic_lpmf(ints k | vector eta, vectors c)

The log ordered logistic probability mass of k given linear predictors eta, and cutpoints c.

Available since 2.18

real ordered_logistic_lupmf(ints k | vector eta, vectors c)

The log ordered logistic probability mass of k given linear predictors eta, and cutpoints c dropping constant additive terms.

Available since 2.25

int ordered_logistic_rng(real eta, vector c)

Generate an ordered logistic variate with linear predictor eta and cutpoints c; may only be used in transformed data and generated quantities blocks

Available since 2.0

17.10. Ordered logistic generalized linear model (ordinal regression)

Probability mass function

If $N, M, K \in \mathbb{N}$ with $N, M > 0$, $K > 2$, $c \in \mathbb{R}^{K-1}$ such that $c_k < c_{k+1}$ for $k \in \{1, \dots, K-2\}$, and $x \in \mathbb{R}^{N \times M}$, $\beta \in \mathbb{R}^M$, then for $y \in \{1, \dots, K\}^N$,

$\text{OrderedLogisticGLM}(y | x, \beta, c)$

$$\begin{aligned}
 &= \prod_{1 \leq i \leq N} \text{OrderedLogistic}(y_i | x_i \cdot \beta, c) \\
 &= \prod_{1 \leq i \leq N} \begin{cases} 1 - \text{logit}^{-1}(x_i \cdot \beta - c_1) & \text{if } y = 1, \\ \text{logit}^{-1}(x_i \cdot \beta - c_{y-1}) - \text{logit}^{-1}(x_i \cdot \beta - c_y) & \text{if } 1 < y < K, \text{ and} \\ \text{logit}^{-1}(x_i \cdot \beta - c_{K-1}) - 0 & \text{if } y = K. \end{cases}
 \end{aligned}$$

The $k = K$ case is written with the redundant subtraction of zero to illustrate the parallelism of the cases; the $y = 1$ and $y = K$ edge cases can be subsumed into the general definition by setting $c_0 = -\infty$ and $c_K = +\infty$ with $\text{logit}^{-1}(-\infty) = 0$ and $\text{logit}^{-1}(\infty) = 1$.

Distribution statement

$y \sim \text{ordered_logistic_glm}(x, \beta, c)$

Increment target log probability density with `ordered_logistic_lupmf(y | x, beta, c)`.

Available since 2.23

Stan functions

```
real ordered_logistic_glm_lpmf(int y | row_vector x, vector beta,
vector c)
```

The log ordered logistic probability mass of y, given linear predictors $x * \beta$, and cutpoints c. The cutpoints c must be ordered.

Available since 2.23

```
real ordered_logistic_glm_lupmf(int y | row_vector x, vector beta,
vector c)
```

The log ordered logistic probability mass of y, given linear predictors $x * \beta$, and cutpoints c dropping constant additive terms. The cutpoints c must be ordered.

Available since 2.25

```
real ordered_logistic_glm_lpmf(int y | matrix x, vector beta, vec-
tor c)
```

The log ordered logistic probability mass of y, given linear predictors $x * \beta$, and cutpoints c. The cutpoints c must be ordered.

Available since 2.23

```
real ordered_logistic_glm_lupmf(int y | matrix x, vector beta, vec-
tor c)
```

The log ordered logistic probability mass of y, given linear predictors $x * \beta$, and cutpoints c dropping constant additive terms. The cutpoints c must be ordered.

Available since 2.25

```
real ordered_logistic_glm_lpmf(array[] int y | row_vector x, vector
beta, vector c)
```

The log ordered logistic probability mass of y, given linear predictors $x * \beta$, and cutpoints c. The cutpoints c must be ordered.

Available since 2.23

```
real ordered_logistic_glm_lupmf(array[] int y | row_vector x, vec-
tor beta, vector c)
```

The log ordered logistic probability mass of y, given linear predictors $x * \beta$, and cutpoints c dropping constant additive terms. The cutpoints c must be ordered.

Available since 2.25

```
real      ordered_logistic_glm_lpmf(array[] int y | matrix x, vector
beta, vector c)
```

The log ordered logistic probability mass of y, given linear predictors x * beta, and cutpoints c. The cutpoints c must be ordered.

Available since 2.23

```
real      ordered_logistic_glm_lupmf(array[] int y | matrix x, vector
beta, vector c)
```

The log ordered logistic probability mass of y, given linear predictors x * beta, and cutpoints c dropping constant additive terms. The cutpoints c must be ordered.

Available since 2.25

17.11. Ordered probit distribution

Probability mass function

If $K \in \mathbb{N}$ with $K > 2$, $c \in \mathbb{R}^{K-1}$ such that $c_k < c_{k+1}$ for $k \in \{1, \dots, K-2\}$, and $\eta \in \mathbb{R}$, then for $k \in \{1, \dots, K\}$,

$$\text{OrderedProbit}(k | \eta, c) = \begin{cases} 1 - \Phi(\eta - c_1) & \text{if } k = 1, \\ \Phi(\eta - c_{k-1}) - \Phi(\eta - c_k) & \text{if } 1 < k < K, \text{ and} \\ \Phi(\eta - c_{K-1}) - 0 & \text{if } k = K. \end{cases}$$

The $k = K$ case is written with the redundant subtraction of zero to illustrate the parallelism of the cases; the $k = 1$ and $k = K$ edge cases can be subsumed into the general definition by setting $c_0 = -\infty$ and $c_K = +\infty$ with $\Phi(-\infty) = 0$ and $\Phi(\infty) = 1$.

Distribution statement

```
k ~ ordered_probit(eta, c)
```

Increment target log probability density with `ordered_probit_lupmf(k | eta, c)`.

Available since 2.19

Stan functions

```
real ordered_probit_lpmf(ints k | vector eta, vectors c)
```

The log ordered probit probability mass of k given linear predictors eta, and cutpoints c.

Available since 2.18

```
real ordered_probit_lupmf(ints k | vector eta, vectors c)
```

The log ordered probit probability mass of k given linear predictors eta, and cutpoints c dropping constant additive terms.

Available since 2.25

```
real ordered_probit_lpmf(ints k | real eta, vectors c)
```

The log ordered probit probability mass of k given linear predictor eta, and cutpoints c.

Available since 2.19

```
real ordered_probit_lupmf(ints k | real eta, vectors c)
```

The log ordered probit probability mass of k given linear predictor eta, and cutpoints c dropping constant additive terms.

Available since 2.19

```
int ordered_probit_rng(real eta, vector c)
```

Generate an ordered probit variate with linear predictor eta and cutpoints c; may only be used in transformed data and generated quantities blocks

Available since 2.18

18. Unbounded Discrete Distributions

The unbounded discrete distributions have support over the natural numbers (i.e., the non-negative integers).

18.1. Negative binomial distribution

For the negative binomial distribution Stan uses the parameterization described in Gelman et al. (2013). For alternative parameterizations, see section [negative binomial](#) `glm`.

Probability mass function

If $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}^+$, then for $n \in \mathbb{N}$,

$$\text{NegBinomial}(n \mid \alpha, \beta) = \binom{n + \alpha - 1}{\alpha - 1} \left(\frac{\beta}{\beta + 1} \right)^\alpha \left(\frac{1}{\beta + 1} \right)^n.$$

The mean and variance of a random variable $n \sim \text{NegBinomial}(\alpha, \beta)$ are given by

$$\mathbb{E}[n] = \frac{\alpha}{\beta} \quad \text{and} \quad \text{Var}[n] = \frac{\alpha}{\beta^2}(\beta + 1).$$

Distribution statement

`n ~ neg_binomial(alpha, beta)`

Increment target log probability density with `neg_binomial_lupmf(n | alpha, beta)`.

Available since 2.0

Stan functions

`real neg_binomial_lpmf(ints n | reals alpha, reals beta)`

The log negative binomial probability mass of n given shape α and inverse scale β

Available since 2.12

`real neg_binomial_lupmf(ints n | reals alpha, reals beta)`

The log negative binomial probability mass of n given shape α and inverse scale β dropping constant additive terms

Available since 2.25

real neg_binomial_cdf(ints n | reals alpha, reals beta)

The negative binomial cumulative distribution function of n given shape alpha and inverse scale beta

Available since 2.0

real neg_binomial_lcdf(ints n | reals alpha, reals beta)

The log of the negative binomial cumulative distribution function of n given shape alpha and inverse scale beta

Available since 2.12

real neg_binomial_lccdf(ints n | reals alpha, reals beta)

The log of the negative binomial complementary cumulative distribution function of n given shape alpha and inverse scale beta

Available since 2.12

R neg_binomial_rng(reals alpha, reals beta)

Generate a negative binomial variate with shape alpha and inverse scale beta; may only be used in transformed data and generated quantities blocks. alpha / beta must be less than 2^{29} . For a description of argument and return types, see section [vectorized function signatures](#).

Available since 2.18

18.2. Negative binomial distribution (alternative parameterization)

Stan also provides an alternative parameterization of the negative binomial distribution directly using a mean (i.e., location) parameter and a parameter that controls overdispersion relative to the square of the mean. Section [combinatorial functions](#), below, provides a second alternative parameterization directly in terms of the log mean.

Probability mass function

The first parameterization is for $\mu \in \mathbb{R}^+$ and $\phi \in \mathbb{R}^+$, which for $n \in \mathbb{N}$ is defined as

$$\text{NegBinomial2}(n | \mu, \phi) = \binom{n + \phi - 1}{n} \left(\frac{\mu}{\mu + \phi} \right)^n \left(\frac{\phi}{\mu + \phi} \right)^\phi.$$

The mean and variance of a random variable $n \sim \text{NegBinomial2}(n \mid \mu, \phi)$ are

$$\mathbb{E}[n] = \mu \quad \text{and} \quad \text{Var}[n] = \mu + \frac{\mu^2}{\phi}.$$

Recall that Poisson(μ) has variance μ , so $\mu^2/\phi > 0$ is the additional variance of the negative binomial above that of the Poisson with mean μ . So the inverse of parameter ϕ controls the overdispersion, scaled by the square of the mean, μ^2 .

Distribution statement

`n ~ neg_binomial_2(mu, phi)`

Increment target log probability density with `neg_binomial_2_lupmf(n | mu, phi)`.

Available since 2.3

Stan functions

`real neg_binomial_2_lpmf(ints n | reals mu, reals phi)`

The log negative binomial probability mass of n given location `mu` and precision `phi`.

Available since 2.20

`real neg_binomial_2_lupmf(ints n | reals mu, reals phi)`

The log negative binomial probability mass of n given location `mu` and precision `phi` dropping constant additive terms.

Available since 2.25

`real neg_binomial_2_cdf(ints n | reals mu, reals phi)`

The negative binomial cumulative distribution function of n given location `mu` and precision `phi`.

Available since 2.6

`real neg_binomial_2_lcdf(ints n | reals mu, reals phi)`

The log of the negative binomial cumulative distribution function of n given location `mu` and precision `phi`.

Available since 2.12

`real neg_binomial_2_lccdf(ints n | reals mu, reals phi)`

The log of the negative binomial complementary cumulative distribution function of n given location `mu` and precision `phi`.

Available since 2.12

R **neg_binomial_2_rng**(reals mu, reals phi)

Generate a negative binomial variate with location mu and precision phi; may only be used in transformed data and generated quantities blocks. mu must be less than 2^{29} . For a description of argument and return types, see section [vectorized function signatures](#).

Available since 2.18

18.3. Negative binomial distribution (log alternative parameterization)

Related to the parameterization in section [negative binomial, alternative parameterization](#), the following parameterization uses a log mean parameter $\eta = \log(\mu)$, defined for $\eta \in \mathbb{R}$, $\phi \in \mathbb{R}^+$, so that for $n \in \mathbb{N}$,

$$\text{NegBinomial2Log}(n | \eta, \phi) = \text{NegBinomial2}(n | \exp(\eta), \phi).$$

This alternative may be used for sampling, as a function, and for random number generation, but as of yet, there are no CDFs implemented for it. This is especially useful for log-linear negative binomial regressions.

Distribution statement

n ~ **neg_binomial_2_log**(eta, phi)

Increment target log probability density with `neg_binomial_2_log_lupmf(n | eta, phi)`.

Available since 2.3

Stan functions

real **neg_binomial_2_log_lpmf**(ints n | reals eta, reals phi)

The log negative binomial probability mass of n given log-location eta and inverse overdispersion parameter phi.

Available since 2.20

real **neg_binomial_2_log_lupmf**(ints n | reals eta, reals phi)

The log negative binomial probability mass of n given log-location eta and inverse overdispersion parameter phi dropping constant additive terms.

Available since 2.25

R **neg_binomial_2_log_rng**(reals eta, reals phi)

Generate a negative binomial variate with log-location eta and inverse overdisper-

18.4. NEGATIVE-BINOMIAL-2-LOG GENERALIZED LINEAR MODEL (NEGATIVE BINOMIAL REGRESSION)

sion control `phi`; may only be used in transformed data and generated quantities blocks. `eta` must be less than $29 \log 2$. For a description of argument and return types, see section [vectorized function signatures](#).

Available since 2.18

18.4. Negative-binomial-2-log generalized linear model (negative binomial regression)

Stan also supplies a single function for a generalized linear model with negative binomial distribution and log link function, i.e. a function for a negative binomial regression. This provides a more efficient implementation of negative binomial regression than a manually written regression in terms of a negative binomial distribution and matrix multiplication.

Probability mass function

If $x \in \mathbb{R}^{n \times m}$, $\alpha \in \mathbb{R}^n$, $\beta \in \mathbb{R}^m$, $\phi \in \mathbb{R}^+$, then for $y \in \mathbb{N}^n$,

$$\text{NegBinomial2LogGLM}(y \mid x, \alpha, \beta, \phi) = \prod_{1 \leq i \leq n} \text{NegBinomial2}(y_i \mid \exp(\alpha_i + x_i \cdot \beta), \phi).$$

Distribution statement

`y ~ neg_binomial_2_log_glm(x, alpha, beta, phi)`

Increment target log probability density with `neg_binomial_2_log_glm_lupmf(y | x, alpha, beta, phi)`.

Available since 2.19

Stan functions

```
real neg_binomial_2_log_glm_lpmf(int y | matrix x, real alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location $\alpha + x \cdot \beta$ and inverse overdispersion parameter ϕ .

Available since 2.23

```
real neg_binomial_2_log_glm_lupmf(int y | matrix x, real alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location $\alpha + x \cdot \beta$ and inverse overdispersion parameter ϕ dropping constant additive terms.

Available since 2.25

```
real    neg_binomial_2_log_glm_lpmf(int y | matrix x, vector alpha,
vector beta, real phi)
```

The log negative binomial probability mass of y given log-location $\alpha + x * \beta$ and inverse overdispersion parameter ϕ .

Available since 2.23

```
real    neg_binomial_2_log_glm_lupmf(int y | matrix x, vector alpha,
vector beta, real phi)
```

The log negative binomial probability mass of y given log-location $\alpha + x * \beta$ and inverse overdispersion parameter ϕ dropping constant additive terms.

Available since 2.25

```
real    neg_binomial_2_log_glm_lpmf(array[] int y | row_vector x, real
alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location $\alpha + x * \beta$ and inverse overdispersion parameter ϕ .

Available since 2.23

```
real    neg_binomial_2_log_glm_lupmf(array[] int y | row_vector x,
real alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location $\alpha + x * \beta$ and inverse overdispersion parameter ϕ dropping constant additive terms.

Available since 2.25

```
real    neg_binomial_2_log_glm_lpmf(array[] int y | row_vector x, vec-
tor alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location $\alpha + x * \beta$ and inverse overdispersion parameter ϕ .

Available since 2.23

```
real    neg_binomial_2_log_glm_lupmf(array[] int y | row_vector x,
vector alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location $\alpha + x * \beta$ and inverse overdispersion parameter ϕ dropping constant additive terms.

Available since 2.25

```
real    neg_binomial_2_log_glm_lpmf(array[] int y | matrix x, real al-
pha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location alpha + x * beta and inverse overdispersion parameter phi.

Available since 2.18

```
real neg_binomial_2_log_glm_lupmf(array[] int y | matrix x, real
alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location alpha + x * beta and inverse overdispersion parameter phi dropping constant additive terms.

Available since 2.25

```
real neg_binomial_2_log_glm_lpmf(array[] int y | matrix x, vector
alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location alpha + x * beta and inverse overdispersion parameter phi.

Available since 2.18

```
real neg_binomial_2_log_glm_lupmf(array[] int y | matrix x, vector
alpha, vector beta, real phi)
```

The log negative binomial probability mass of y given log-location alpha + x * beta and inverse overdispersion parameter phi dropping constant additive terms.

Available since 2.25

18.5. Poisson distribution

Probability mass function

If $\lambda \in \mathbb{R}^+$, then for $n \in \mathbb{N}$,

$$\text{Poisson}(n|\lambda) = \frac{1}{n!} \lambda^n \exp(-\lambda).$$

Distribution statement

`n ~ poisson(lambda)`

Increment target log probability density with `poisson_lupmf(n | lambda)`.

Available since 2.0

Stan functions

```
real poisson_lpmf(ints n | reals lambda)
```

The log Poisson probability mass of n given rate lambda

Available since 2.12

real poisson_lupmf(ints n | reals lambda)

The log Poisson probability mass of n given rate lambda dropping constant additive terms

Available since 2.25

real poisson_cdf(ints n | reals lambda)

The Poisson cumulative distribution function of n given rate lambda

Available since 2.0

real poisson_lcdf(ints n | reals lambda)

The log of the Poisson cumulative distribution function of n given rate lambda

Available since 2.12

real poisson_lccdf(ints n | reals lambda)

The log of the Poisson complementary cumulative distribution function of n given rate lambda

Available since 2.12

R poisson_rng(reals lambda)

Generate a Poisson variate with rate lambda; may only be used in transformed data and generated quantities blocks. lambda must be less than 2^{30} . For a description of argument and return types, see section [vectorized function signatures](#).

Available since 2.18

18.6. Poisson distribution, log parameterization

Stan also provides a parameterization of the Poisson using the log rate $\alpha = \log \lambda$ as a parameter. This is useful for log-linear Poisson regressions so that the predictor does not need to be exponentiated and passed into the standard Poisson probability function.

Probability mass function

If $\alpha \in \mathbb{R}$, then for $n \in \mathbb{N}$,

$$\text{PoissonLog}(n|\alpha) = \frac{1}{n!} \exp(n\alpha - \exp(\alpha)).$$

Distribution statement

n ~ poisson_log(alpha)

Increment target log probability density with `poisson_log_lupmf(n | alpha)`.

Available since 2.0

Stan functions

real **poisson_log_lpmf**(int n | real alpha)

The log Poisson probability mass of n given log rate alpha

Available since 2.12

real **poisson_log_lupmf**(int n | real alpha)

The log Poisson probability mass of n given log rate alpha dropping constant additive terms

Available since 2.25

R **poisson_log_rng**(real alpha)

Generate a Poisson variate with log rate alpha; may only be used in transformed data and generated quantities blocks. alpha must be less than $30 \log 2$. For a description of argument and return types, see section [vectorized function signatures](#).

Available since 2.18

18.7. Poisson-log generalized linear model (Poisson regression)

Stan also supplies a single function for a generalized linear model with Poisson distribution and log link function, i.e. a function for a Poisson regression. This provides a more efficient implementation of Poisson regression than a manually written regression in terms of a Poisson distribution and matrix multiplication.

Probability mass function

If $x \in \mathbb{R}^{n \cdot m}$, $\alpha \in \mathbb{R}^n$, $\beta \in \mathbb{R}^m$, then for $y \in \mathbb{N}^n$,

$$\text{PoissonLogGLM}(y|x, \alpha, \beta) = \prod_{1 \leq i \leq n} \text{Poisson}(y_i | \exp(\alpha_i + x_i \cdot \beta)).$$

Distribution statement

$y \sim \text{poisson_log_glm}(x, \alpha, \beta)$

Increment target log probability density with `poisson_log_glm_lupmf(y | x, alpha, beta)`.

Available since 2.19

Stan functions

real **poisson_log_glm_lpmf**(int y | matrix x, real alpha, vector beta)

The log Poisson probability mass of y given the log-rate alpha + x * beta.

Available since 2.23

```
real    poisson_log_glm_lupmf(int y | matrix x, real alpha, vector
beta)
```

The log Poisson probability mass of y given the log-rate alpha + x * beta dropping constant additive terms.

Available since 2.25

```
real    poisson_log_glm_lpmf(int y | matrix x, vector alpha, vector
beta)
```

The log Poisson probability mass of y given the log-rate alpha + x * beta.

Available since 2.23

```
real    poisson_log_glm_lupmf(int y | matrix x, vector alpha, vector
beta)
```

The log Poisson probability mass of y given the log-rate alpha + x * beta dropping constant additive terms.

Available since 2.25

```
real poisson_log_glm_lpmf(array[] int y | row_vector x, real alpha,
vector beta)
```

The log Poisson probability mass of y given the log-rate alpha + x * beta.

Available since 2.23

```
real    poisson_log_glm_lupmf(array[] int y | row_vector x, real al-
pha, vector beta)
```

The log Poisson probability mass of y given the log-rate alpha + x * beta dropping constant additive terms.

Available since 2.25

```
real poisson_log_glm_lpmf(array[] int y | row_vector x, vector al-
pha, vector beta)
```

The log Poisson probability mass of y given the log-rate alpha + x * beta.

Available since 2.23

```
real poisson_log_glm_lupmf(array[] int y | row_vector x, vector al-
pha, vector beta)
```

The log Poisson probability mass of y given the log-rate $\alpha + x * \beta$ dropping constant additive terms.

Available since 2.25

```
real    poisson_log_glm_lpmf(array[] int y | matrix x, real alpha,
vector beta)
```

The log Poisson probability mass of y given the log-rate $\alpha + x * \beta$.

Available since 2.18

```
real    poisson_log_glm_lupmf(array[] int y | matrix x, real alpha,
vector beta)
```

The log Poisson probability mass of y given the log-rate $\alpha + x * \beta$ dropping constant additive terms.

Available since 2.25

```
real    poisson_log_glm_lpmf(array[] int y | matrix x, vector alpha,
vector beta)
```

The log Poisson probability mass of y given the log-rate $\alpha + x * \beta$.

Available since 2.18

```
real    poisson_log_glm_lupmf(array[] int y | matrix x, vector alpha,
vector beta)
```

The log Poisson probability mass of y given the log-rate $\alpha + x * \beta$ dropping constant additive terms.

Available since 2.25

18.8. Beta negative binomial distribution

Probability mass function

If $r \in \mathbb{R}^+$, $\alpha \in \mathbb{R}^+$, and $\beta \in \mathbb{R}^+$, then for $n \in \mathbb{N}$,

$$\text{BetaNegBinomial}(n|r, \alpha, \beta) = \frac{\Gamma(n+r)}{n! \Gamma(r)} \frac{B(\beta+n, \alpha+r)}{B(\beta, \alpha)}.$$

Distribution statement

```
n ~ beta_neg_binomial(r, alpha, beta)
```

Increment target log probability density with `beta_neg_binomial_lupmf(n | r, alpha, beta)`.

Available since 2.36

Stan functions

```
real beta_neg_binomial_lpmf(ints n | reals r, reals alpha, reals beta)
```

The log beta negative binomial probability mass of n given parameters r, alpha and beta.

Available since 2.36

```
real beta_neg_binomial_lupmf(ints n | reals r, reals alpha, reals beta)
```

The log beta negative binomial probability mass of n given parameters r, alpha and beta dropping constant additive terms.

Available since 2.36

```
real beta_neg_binomial_cdf(ints n | reals r, reals alpha, reals beta)
```

The beta negative binomial cumulative distribution function of n given parameters r, alpha and beta.

Available since 2.36

```
real beta_neg_binomial_lcdf(ints n | reals r, reals alpha, reals beta)
```

The log of the beta negative binomial cumulative distribution function of n given parameters r, alpha and beta.

Available since 2.36

```
real beta_neg_binomial_lccdf(ints n | reals r, reals alpha, reals beta)
```

The log of the beta negative binomial complementary cumulative distribution function of n given parameters r, alpha and beta.

Available since 2.36

R `beta_neg_binomial_rng`(reals r, reals alpha, reals beta)

Generate a beta negative binomial variate with parameters r, alpha and beta; may only be used in transformed data and generated quantities blocks. $r \cdot \text{beta} / (\text{alpha} - 1)$ must be less than 2^{29} . For a description of argument and return types, see section [vectorized function signatures](#).

Available since 2.36

19. Multivariate Discrete Distributions

The multivariate discrete distributions are over multiple integer values, which are expressed in Stan as arrays.

19.1. Multinomial distribution

Probability mass function

If $K \in \mathbb{N}$, $N \in \mathbb{N}$, and $\theta \in K$ -simplex, then for $y \in \mathbb{N}^K$ such that $\sum_{k=1}^K y_k = N$,

$$\text{Multinomial}(y|\theta) = \binom{N}{y_1, \dots, y_K} \prod_{k=1}^K \theta_k^{y_k},$$

where the multinomial coefficient is defined by

$$\binom{N}{y_1, \dots, y_K} = \frac{N!}{\prod_{k=1}^K y_k!}.$$

Distribution statement

`y ~ multinomial(theta)`

Increment target log probability density with `multinomial_lupmf(y | theta)`.

Available since 2.0

Stan functions

`real multinomial_lpmf(array[] int y | vector theta)`

The log multinomial probability mass function with outcome array `y` of size K given the K -simplex distribution parameter `theta` and (implicit) total count `N = sum(y)`

Available since 2.12

`real multinomial_lupmf(array[] int y | vector theta)`

The log multinomial probability mass function with outcome array `y` of size K given the K -simplex distribution parameter `theta` and (implicit) total count `N = sum(y)` dropping constant additive terms

Available since 2.25

```
array[] int multinomial_rng(vector theta, int N)
```

Generate a multinomial variate with simplex distribution parameter theta and total count N ; may only be used in transformed data and generated quantities blocks

Available since 2.8

19.2. Multinomial distribution, logit parameterization

Stan also provides a version of the multinomial probability mass function distribution with the K -simplex for the event count probabilities per category given on the unconstrained logistic scale.

Probability mass function

If $K \in \mathbb{N}$, $N \in \mathbb{N}$, and $\text{softmax}(\theta) \in K\text{-simplex}$, then for $y \in \mathbb{N}^K$ such that $\sum_{k=1}^K y_k = N$,

$$\begin{aligned} \text{MultinomialLogit}(y \mid \gamma) &= \text{Multinomial}(y \mid \text{softmax}(\gamma)) \\ &= \binom{N}{y_1, \dots, y_K} \prod_{k=1}^K [\text{softmax}(\gamma_k)]^{y_k}, \end{aligned}$$

where the multinomial coefficient is defined by

$$\binom{N}{y_1, \dots, y_K} = \frac{N!}{\prod_{k=1}^K y_k!}.$$

Distribution statement

```
y ~ multinomial_logit(gamma)
```

Increment target log probability density with `multinomial_logit_lupmf(y | gamma)`.

Available since 2.24

Stan functions

```
real multinomial_logit_lpmf(array[] int y | vector gamma)
```

The log multinomial probability mass function with outcome array y of size K given the log K -simplex distribution parameter γ and (implicit) total count $N = \text{sum}(y)$

Available since 2.24

```
real multinomial_logit_lupmf(array[] int y | vector gamma)
```

The log multinomial probability mass function with outcome array y of size K given

the log K -simplex distribution parameter γ and (implicit) total count $N = \text{sum}(y)$ dropping constant additive terms

Available since 2.25

`array[] int multinomial_logit_rng(vector gamma, int N)`

Generate a variate from a multinomial distribution with probabilities $\text{softmax}(\gamma)$ and total count N ; may only be used in transformed data and generated quantities blocks.

Available since 2.24

19.3. Dirichlet-multinomial distribution

Stan also provides the Dirichlet-multinomial distribution, which generalizes the Beta-binomial distribution to more than two categories. As such, it is an overdispersed version of the multinomial distribution.

Probability mass function

If $K \in \mathbb{N}$, $N \in \mathbb{N}$, and $\alpha \in \mathbb{R}_+^K$, then for $y \in \mathbb{N}^K$ such that $\sum_{k=1}^K y_k = N$, the PMF of the Dirichlet-multinomial distribution is defined as

$$\text{DirMult}(y|\theta) = \frac{\Gamma(\alpha_0)\Gamma(N+1)}{\Gamma(N+\alpha_0)} \prod_{k=1}^K \frac{\Gamma(y_k + \alpha_k)}{\Gamma(\alpha_k)\Gamma(y_k + 1)},$$

where α_0 is defined as $\alpha_0 = \sum_{k=1}^K \alpha_k$.

Distribution statement

`y ~ dirichlet_multinomial(alpha)`

Increment target log probability density with `dirichlet_multinomial_lupmf(y | alpha)`.

Available since 2.34

Stan functions

`real dirichlet_multinomial_lpmf(array[] int y | vector alpha)`

The log multinomial probability mass function with outcome array `y` with K elements given the positive K -vector distribution parameter `alpha` and (implicit) total count $N = \text{sum}(y)$.

Available since 2.34

`real dirichlet_multinomial_lupmf(array[] int y | vector alpha)`

The log multinomial probability mass function with outcome array `y` with K ele-

ments, given the positive K -vector distribution parameter `alpha` and (implicit) total count $N = \text{sum}(y)$ dropping constant additive terms.

Available since 2.34

```
array[] int dirichlet_multinomial_rng(vector alpha, int N)
```

Generate a multinomial variate with positive vector distribution parameter `alpha` and total count `N`; may only be used in transformed data and generated quantities blocks. This is equivalent to `multinomial_rng(dirichlet_rng(alpha), N)`.

Available since 2.34

Part III

Continuous Distributions

20. Unbounded Continuous Distributions

The unbounded univariate continuous probability distributions have support on all real numbers.

20.1. Normal distribution

Probability density function

If $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}$,

$$\text{Normal}(y|\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2\right).$$

Distribution statement

`y ~ normal(mu, sigma)`

Increment target log probability density with `normal_lupdf(y | mu, sigma)`.

Available since 2.0

Stan functions

`real normal_lpdf(reals y | reals mu, reals sigma)`

The log of the normal density of y given location μ and scale σ

Available since 2.12

`real normal_lupdf(reals y | reals mu, reals sigma)`

The log of the normal density of y given location μ and scale σ dropping constant additive terms.

Available since 2.25

`real normal_cdf(reals y | reals mu, reals sigma)`

The cumulative normal distribution of y given location μ and scale σ ; `normal_cdf` will underflow to 0 for $\frac{y-\mu}{\sigma}$ below -37.5 and overflow to 1 for $\frac{y-\mu}{\sigma}$ above 8.25; the function `Phi_approx` is more robust in the tails, but must be scaled and translated for anything other than a standard normal.

Available since 2.0

real `normal_lcdf`(reals y | reals mu, reals sigma)

The log of the cumulative normal distribution of y given location mu and scale sigma; `normal_lcdf` will underflow to $-\infty$ for $\frac{y-\mu}{\sigma}$ below -37.5 and overflow to 0 for $\frac{y-\mu}{\sigma}$ above 8.25; `log(Phi_approx(...))` is more robust in the tails, but must be scaled and translated for anything other than a standard normal.

Available since 2.12

real `normal_lccdf`(reals y | reals mu, reals sigma)

The log of the complementary cumulative normal distribution of y given location mu and scale sigma; `normal_lccdf` will overflow to 0 for $\frac{y-\mu}{\sigma}$ below -37.5 and underflow to $-\infty$ for $\frac{y-\mu}{\sigma}$ above 8.25; `log1m(Phi_approx(...))` is more robust in the tails, but must be scaled and translated for anything other than a standard normal.

Available since 2.15

R `normal_rng`(reals mu, reals sigma)

Generate a normal variate with location mu and scale sigma; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

Standard normal distribution

The standard normal distribution is so-called because its parameters are the units for their respective operations—the location (mean) is zero and the scale (standard deviation) one. The standard normal is parameter-free, and the unit parameters allow considerable simplification of the expression for the density.

$$\text{StdNormal}(y) = \text{Normal}(y | 0, 1) = \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-y^2}{2}\right).$$

Up to a proportion on the log scale, where Stan computes,

$$\log \text{Normal}(y | 0, 1) = \frac{-y^2}{2} + \text{const.}$$

With no logarithm, no subtraction, and no division by a parameter, the standard normal log density is much more efficient to compute than the normal log density with constant location 0 and scale 1.

Distribution statement

```
y ~ std_normal()
```

Increment target log probability density with `std_normal_lupdf(y)`.

Available since 2.19

Stan functions

```
real std_normal_lpdf(reals y)
```

The standard normal (location zero, scale one) log probability density of y .

Available since 2.18

```
real std_normal_lupdf(reals y)
```

The standard normal (location zero, scale one) log probability density of y dropping constant additive terms.

Available since 2.25

```
real std_normal_cdf(reals y)
```

The cumulative standard normal distribution of y ; `std_normal_cdf` will underflow to 0 for y below -37.5 and overflow to 1 for y above 8.25; the function `Phi_approx` is more robust in the tails.

Available since 2.21

```
real std_normal_lcdf(reals y)
```

The log of the cumulative standard normal distribution of y ; `std_normal_lcdf` will underflow to $-\infty$ for y below -37.5 and overflow to 0 for y above 8.25; `log(Phi_approx(...))` is more robust in the tails.

Available since 2.21

```
real std_normal_lccdf(reals y)
```

The log of the complementary cumulative standard normal distribution of y ; `std_normal_lccdf` will overflow to 0 for y below -37.5 and underflow to $-\infty$ for y above 8.25; `log1m(Phi_approx(...))` is more robust in the tails.

Available since 2.21

```
R std_normal_qf(T x)
```

Returns the value of the inverse standard normal cdf Φ^{-1} at the specified quantile x . The `std_normal_qf` is equivalent to the `inv_Phi` function.

Available since 2.31

R `std_normal_log_qf`(T x)

Return the value of the inverse standard normal cdf Φ^{-1} evaluated at the log of the specified quantile x . This function is equivalent to `std_normal_qf(exp(x))` but is more numerically stable.

Available since 2.31

real `std_normal_rng`()

Generate a normal variate with location zero and scale one; may only be used in transformed data and generated quantities blocks.

Available since 2.21

20.2. Normal-id generalized linear model (linear regression)

Stan also supplies a single function for a generalized linear model with normal distribution and identity link function, i.e. a function for a linear regression. This provides a more efficient implementation of linear regression than a manually written regression in terms of a normal distribution and matrix multiplication.

Probability distribution function

If $x \in \mathbb{R}^{n \cdot m}$, $\alpha \in \mathbb{R}^n$, $\beta \in \mathbb{R}^m$, $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}^n$,

$$\text{NormalIdGLM}(y|x, \alpha, \beta, \sigma) = \prod_{1 \leq i \leq n} \text{Normal}(y_i | \alpha_i + x_i \cdot \beta, \sigma).$$

Distribution statement

`y ~ normal_id_glm(x, alpha, beta, sigma)`

Increment target log probability density with `normal_id_glm_lupdf(y | x, alpha, beta, sigma)`.

Available since 2.19

Stan functions

`real normal_id_glm_lpdf(real y | matrix x, real alpha, vector beta, real sigma)`

The log normal probability density of y given location $\alpha + x \cdot \beta$ and scale σ .

Available since 2.29

`real normal_id_glm_lupdf(real y | matrix x, real alpha, vector beta, real sigma)`

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.29

```
real      normal_id_glm_lpdf(real y | matrix x, vector alpha, vector
beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.29

```
real      normal_id_glm_lupdf(real y | matrix x, vector alpha, vector
beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.29

```
real      normal_id_glm_lpdf(real y | matrix x, real alpha, vector beta,
vector sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.23

```
real      normal_id_glm_lupdf(real y | matrix x, real alpha, vector
beta, vector sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.25

```
real      normal_id_glm_lpdf(real y | matrix x, vector alpha, vector
beta, vector sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.23

```
real      normal_id_glm_lupdf(real y | matrix x, vector alpha, vector
beta, vector sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.25

```
real normal_id_glm_lpdf(vector y | row_vector x, real alpha, vector
beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.29

```
real normal_id_glm_lupdf(vector y | row_vector x, real alpha, vec-
tor beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.29

```
real normal_id_glm_lpdf(vector y | row_vector x, vector alpha, vec-
tor beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.29

```
real      normal_id_glm_lupdf(vector y | row_vector x, vector alpha,
vector beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.29

```
real      normal_id_glm_lpdf(vector y | matrix x, real alpha, vector
beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.23

```
real      normal_id_glm_lupdf(vector y | matrix x, real alpha, vector
beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.23

```
real      normal_id_glm_lpdf(vector y | matrix x, vector alpha, vector
beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.23

```
real normal_id_glm_lupdf(vector y | matrix x, vector alpha, vector  
beta, real sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.23

```
real normal_id_glm_lpdf(vector y | matrix x, real alpha, vector  
beta, vector sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.30

```
real normal_id_glm_lupdf(vector y | matrix x, real alpha, vector  
beta, vector sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.30

```
real normal_id_glm_lpdf(vector y | matrix x, vector alpha, vector  
beta, vector sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ .

Available since 2.30

```
real normal_id_glm_lupdf(vector y | matrix x, vector alpha, vector  
beta, vector sigma)
```

The log normal probability density of y given location $\alpha + x * \beta$ and scale σ dropping constant additive terms.

Available since 2.30

20.3. Exponentially modified normal distribution

Exponentially modified Gaussian describes the distribution of $Z = X + Y$ when X and Y are independent and X is normally distributed (with mean μ and standard deviation σ) and Y is exponentially distributed (with rate λ).

Probability density function

If $\mu \in \mathbb{R}$, $\sigma \in \mathbb{R}^+$, and $\lambda \in \mathbb{R}^+$, then for $y \in \mathbb{R}$,

$$\text{ExpModNormal}(y|\mu, \sigma, \lambda) = \frac{\lambda}{2} \exp\left(\frac{\lambda}{2} (2\mu + \lambda\sigma^2 - 2y)\right) \text{erfc}\left(\frac{\mu + \lambda\sigma^2 - y}{\sqrt{2}\sigma}\right).$$

Distribution statement

`y ~ exp_mod_normal(mu, sigma, lambda)`

Increment target log probability density with `exp_mod_normal_lupdf(y | mu, sigma, lambda)`.

Available since 2.0

Stan functions

`real exp_mod_normal_lpdf(reals y | reals mu, reals sigma, reals lambda)`

The log of the exponentially modified normal density of y given location μ , scale σ , and rate λ

Available since 2.18

`real exp_mod_normal_lupdf(reals y | reals mu, reals sigma, reals lambda)`

The log of the exponentially modified normal density of y given location μ , scale σ , and rate λ dropping constant additive terms

Available since 2.25

`real exp_mod_normal_cdf(reals y | reals mu, reals sigma, reals lambda)`

The exponentially modified normal cumulative distribution function of y given location μ , scale σ , and rate λ

Available since 2.0

`real exp_mod_normal_lcdf(reals y | reals mu, reals sigma, reals lambda)`

The log of the exponentially modified normal cumulative distribution function of y given location μ , scale σ , and rate λ

Available since 2.18

`real exp_mod_normal_lccdf(reals y | reals mu, reals sigma, reals lambda)`

The log of the exponentially modified normal complementary cumulative distribution function of y given location mu, scale sigma, and rate lambda

Available since 2.18

R `exp_mod_normal_rng`(reals mu, reals sigma, reals lambda)

Generate a exponentially modified normal variate with location mu, scale sigma, and rate lambda; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

20.4. Skew normal distribution

Probability density function

If $\xi \in \mathbb{R}$, $\omega \in \mathbb{R}^+$, and $\alpha \in \mathbb{R}$, then for $y \in \mathbb{R}$,

$$\text{SkewNormal}(y | \xi, \omega, \alpha) = \frac{1}{\omega\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{y-\xi}{\omega}\right)^2\right) \left(1 + \text{erf}\left(\alpha\left(\frac{y-\xi}{\omega\sqrt{2}}\right)\right)\right).$$

Distribution statement

`y ~ skew_normal(xi, omega, alpha)`

Increment target log probability density with `skew_normal_lupdf(y | xi, omega, alpha)`.

Available since 2.0

Stan functions

real `skew_normal_lpdf`(reals y | reals xi, reals omega, reals alpha)

The log of the skew normal density of y given location xi, scale omega, and shape alpha

Available since 2.16

real `skew_normal_lupdf`(reals y | reals xi, reals omega, reals alpha)

The log of the skew normal density of y given location xi, scale omega, and shape alpha dropping constant additive terms

Available since 2.25

real `skew_normal_cdf`(reals y | reals xi, reals omega, reals alpha)
 The skew normal distribution function of y given location xi, scale omega, and shape alpha

Available since 2.16

real `skew_normal_lcdf`(reals y | reals xi, reals omega, reals alpha)
 The log of the skew normal cumulative distribution function of y given location xi, scale omega, and shape alpha

Available since 2.18

real `skew_normal_lccdf`(reals y | reals xi, reals omega, reals alpha)

The log of the skew normal complementary cumulative distribution function of y given location xi, scale omega, and shape alpha

Available since 2.18

R `skew_normal_rng`(reals xi, reals omega, real alpha)

Generate a skew normal variate with location xi, scale omega, and shape alpha; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

20.5. Student-t distribution

Probability density function

If $\nu \in \mathbb{R}^+$, $\mu \in \mathbb{R}$, and $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}$,

$$\text{StudentT}(y|\nu, \mu, \sigma) = \frac{\Gamma((\nu+1)/2)}{\Gamma(\nu/2)} \frac{1}{\sqrt{\nu\pi}\sigma} \left(1 + \frac{1}{\nu} \left(\frac{y-\mu}{\sigma}\right)^2\right)^{-(\nu+1)/2}.$$

Distribution statement

`y ~ student_t(nu, mu, sigma)`

Increment target log probability density with `student_t_lupdf(y | nu, mu, sigma)`.

Available since 2.0

Stan functions

real student_t_lpdf(reals y | reals nu, reals mu, reals sigma)

The log of the Student-*t* density of *y* given degrees of freedom *nu*, location *mu*, and scale *sigma*

Available since 2.12

real student_t_lupdf(reals y | reals nu, reals mu, reals sigma)

The log of the Student-*t* density of *y* given degrees of freedom *nu*, location *mu*, and scale *sigma* dropping constant additive terms

Available since 2.25

real student_t_cdf(reals y | reals nu, reals mu, reals sigma)

The Student-*t* cumulative distribution function of *y* given degrees of freedom *nu*, location *mu*, and scale *sigma*

Available since 2.0

real student_t_lcdf(reals y | reals nu, reals mu, reals sigma)

The log of the Student-*t* cumulative distribution function of *y* given degrees of freedom *nu*, location *mu*, and scale *sigma*

Available since 2.12

real student_t_lccdf(reals y | reals nu, reals mu, reals sigma)

The log of the Student-*t* complementary cumulative distribution function of *y* given degrees of freedom *nu*, location *mu*, and scale *sigma*

Available since 2.12

R student_t_rng(reals nu, reals mu, reals sigma)

Generate a Student-*t* variate with degrees of freedom *nu*, location *mu*, and scale *sigma*; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

20.6. Cauchy distribution

Probability density function

If $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}$,

$$\text{Cauchy}(y|\mu, \sigma) = \frac{1}{\pi\sigma} \frac{1}{1 + ((y - \mu)/\sigma)^2}.$$

Distribution statement

`y ~ cauchy(mu, sigma)`

Increment target log probability density with `cauchy_lupdf(y | mu, sigma)`.

Available since 2.0

Stan functions

`real cauchy_lpdf(reals y | reals mu, reals sigma)`

The log of the Cauchy density of y given location mu and scale sigma

Available since 2.12

`real cauchy_lupdf(reals y | reals mu, reals sigma)`

The log of the Cauchy density of y given location mu and scale sigma dropping constant additive terms

Available since 2.25

`real cauchy_cdf(reals y | reals mu, reals sigma)`

The Cauchy cumulative distribution function of y given location mu and scale sigma

Available since 2.0

`real cauchy_lcdf(reals y | reals mu, reals sigma)`

The log of the Cauchy cumulative distribution function of y given location mu and scale sigma

Available since 2.12

`real cauchy_lccdf(reals y | reals mu, reals sigma)`

The log of the Cauchy complementary cumulative distribution function of y given location mu and scale sigma

Available since 2.12

`R cauchy_rng(reals mu, reals sigma)`

Generate a Cauchy variate with location mu and scale sigma; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

20.7. Double exponential (Laplace) distribution

Probability density function

If $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}$,

$$\text{DoubleExponential}(y|\mu, \sigma) = \frac{1}{2\sigma} \exp\left(-\frac{|y - \mu|}{\sigma}\right).$$

Note that the double exponential distribution is parameterized in terms of the scale, in contrast to the exponential distribution (see section [exponential distribution](#)), which is parameterized in terms of inverse scale.

The double-exponential distribution can be defined as a compound exponential-normal distribution (Ding and Blitzstein 2018). Using the inverse scale parameterization for the exponential distribution, and the standard deviation parameterization for the normal distribution, one can write

$$\alpha \sim \text{Exponential}\left(\frac{1}{2\sigma^2}\right)$$

and

$$\beta \mid \alpha \sim \text{Normal}(\mu, \sqrt{\alpha}),$$

then

$$\beta \sim \text{DoubleExponential}(\mu, \sigma).$$

This may be used to code a non-centered parameterization by taking

$$\beta^{\text{raw}} \sim \text{Normal}(0, 1)$$

and defining

$$\beta = \mu + \sqrt{\alpha} \beta^{\text{raw}}.$$

Distribution statement

`y ~ double_exponential(mu, sigma)`

Increment target log probability density with `double_exponential_lupdf(y | mu, sigma)`.

Available since 2.0

Stan functions

real `double_exponential_lpdf`(reals y | reals mu, reals sigma)

The log of the double exponential density of y given location mu and scale sigma

Available since 2.12

real `double_exponential_lupdf`(reals y | reals mu, reals sigma)

The log of the double exponential density of y given location mu and scale sigma dropping constant additive terms

Available since 2.25

real `double_exponential_cdf`(reals y | reals mu, reals sigma)

The double exponential cumulative distribution function of y given location mu and scale sigma

Available since 2.0

real `double_exponential_lcdf`(reals y | reals mu, reals sigma)

The log of the double exponential cumulative distribution function of y given location mu and scale sigma

Available since 2.12

real `double_exponential_lccdf`(reals y | reals mu, reals sigma)

The log of the double exponential complementary cumulative distribution function of y given location mu and scale sigma

Available since 2.12

R `double_exponential_rng`(reals mu, reals sigma)

Generate a double exponential variate with location mu and scale sigma; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

20.8. Logistic distribution

Probability density function

If $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}$,

$$\text{Logistic}(y|\mu, \sigma) = \frac{1}{\sigma} \exp\left(-\frac{y-\mu}{\sigma}\right) \left(1 + \exp\left(-\frac{y-\mu}{\sigma}\right)\right)^{-2}.$$

Distribution statement

`y ~ logistic(mu, sigma)`

Increment target log probability density with `logistic_lupdf(y | mu, sigma)`.

Available since 2.0

Stan functions

`real logistic_lpdf(reals y | reals mu, reals sigma)`

The log of the logistic density of y given location mu and scale sigma

Available since 2.12

`real logistic_lupdf(reals y | reals mu, reals sigma)`

The log of the logistic density of y given location mu and scale sigma dropping constant additive terms

Available since 2.25

`real logistic_cdf(reals y | reals mu, reals sigma)`

The logistic cumulative distribution function of y given location mu and scale sigma

Available since 2.0

`real logistic_lcdf(reals y | reals mu, reals sigma)`

The log of the logistic cumulative distribution function of y given location mu and scale sigma

Available since 2.12

`real logistic_lccdf(reals y | reals mu, reals sigma)`

The log of the logistic complementary cumulative distribution function of y given location mu and scale sigma

Available since 2.12

`R logistic_rng(reals mu, reals sigma)`

Generate a logistic variate with location mu and scale sigma; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

20.9. Gumbel distribution

Probability density function

If $\mu \in \mathbb{R}$ and $\beta \in \mathbb{R}^+$, then for $y \in \mathbb{R}$,

$$\text{Gumbel}(y|\mu, \beta) = \frac{1}{\beta} \exp\left(-\frac{y-\mu}{\beta} - \exp\left(-\frac{y-\mu}{\beta}\right)\right).$$

Distribution statement

`y ~ gumbel(mu, beta)`

Increment target log probability density with `gumbel_lupdf(y | mu, beta)`.

Available since 2.0

Stan functions

`real gumbel_lpdf(reals y | reals mu, reals beta)`

The log of the gumbel density of y given location μ and scale β

Available since 2.12

`real gumbel_lupdf(reals y | reals mu, reals beta)`

The log of the gumbel density of y given location μ and scale β dropping constant additive terms

Available since 2.25

`real gumbel_cdf(reals y | reals mu, reals beta)`

The gumbel cumulative distribution function of y given location μ and scale β

Available since 2.0

`real gumbel_lcdf(reals y | reals mu, reals beta)`

The log of the gumbel cumulative distribution function of y given location μ and scale β

Available since 2.12

`real gumbel_lccdf(reals y | reals mu, reals beta)`

The log of the gumbel complementary cumulative distribution function of y given location μ and scale β

Available since 2.12

`R gumbel_rng(reals mu, reals beta)`

Generate a gumbel variate with location μ and scale β ; may only be used in

transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

20.10. Skew double exponential distribution

Probability density function

If $\mu \in \mathbb{R}$, $\sigma \in \mathbb{R}^+$ and $\tau \in [0, 1]$, then for $y \in \mathbb{R}$,

$$\text{SkewDoubleExponential}(y|\mu, \sigma, \tau) = \frac{2\tau(1-\tau)}{\sigma} \exp \left[-\frac{2}{\sigma} [(1-\tau) I(y < \mu)(\mu - y) + \tau I(y > \mu)(y - \mu)] \right]$$

Distribution statement

```
y ~ skew_double_exponential(mu, sigma, tau)
```

Increment target log probability density with `skew_double_exponential(y | mu, sigma, tau)`

Available since 2.28

Stan functions

```
real skew_double_exponential_lpdf(reals y | reals mu, reals sigma, reals tau)
```

The log of the skew double exponential density of y given location mu, scale sigma and skewness tau

Available since 2.28

```
real skew_double_exponential_lupdf(reals y | reals mu, reals sigma, reals tau)
```

The log of the skew double exponential density of y given location mu, scale sigma and skewness tau dropping constant additive terms

Available since 2.28

```
real skew_double_exponential_cdf(reals y | reals mu, reals sigma, reals tau)
```

The skew double exponential cumulative distribution function of y given location mu, scale sigma and skewness tau

Available since 2.28

```
real skew_double_exponential_lcdf(reals y | reals mu, reals sigma,  
reals tau)
```

The log of the skew double exponential cumulative distribution function of y given location mu, scale sigma and skewness tau

Available since 2.28

```
real skew_double_exponential_lccdf(reals y | reals mu, reals sigma,  
reals tau)
```

The log of the skew double exponential complementary cumulative distribution function of y given location mu, scale sigma and skewness tau

Available since 2.28

```
R skew_double_exponential_rng(reals mu, reals sigma, reals tau)
```

Generate a skew double exponential variate with location mu, scale sigma and skewness tau; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.28

21. Positive Continuous Distributions

The positive continuous probability functions have support on the positive real numbers.

21.1. Lognormal distribution

Probability density function

If $\mu \in \mathbb{R}$ and $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{LogNormal}(y|\mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} \frac{1}{y} \exp\left(-\frac{1}{2} \left(\frac{\log y - \mu}{\sigma}\right)^2\right).$$

Distribution statement

`y ~ lognormal(mu, sigma)`

Increment target log probability density with `lognormal_lupdf(y | mu, sigma)`.

Available since 2.0

Stan functions

`real lognormal_lpdf(reals y | reals mu, reals sigma)`

The log of the lognormal density of y given location mu and scale sigma

Available since 2.12

`real lognormal_lupdf(reals y | reals mu, reals sigma)`

The log of the lognormal density of y given location mu and scale sigma dropping constant additive terms

Available since 2.25

`real lognormal_cdf(reals y | reals mu, reals sigma)`

The cumulative lognormal distribution function of y given location mu and scale sigma

Available since 2.0

`real lognormal_lcdf(reals y | reals mu, reals sigma)`

The log of the lognormal cumulative distribution function of y given location mu and scale sigma

Available since 2.12

real `lognormal_lccdf`(reals y | reals mu, reals sigma)

The log of the lognormal complementary cumulative distribution function of y given location mu and scale sigma

Available since 2.12

R `lognormal_rng`(reals mu, reals sigma)

Generate a lognormal variate with location mu and scale sigma; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.22

21.2. Chi-square distribution

Probability density function

If $\nu \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{ChiSquare}(y|\nu) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} y^{\nu/2-1} \exp\left(-\frac{1}{2}y\right).$$

Distribution statement

`y ~ chi_square(nu)`

Increment target log probability density with `chi_square_lupdf(y | nu)`.

Available since 2.0

Stan functions

real `chi_square_lpdf`(reals y | reals nu)

The log of the Chi-square density of y given degrees of freedom nu

Available since 2.12

real `chi_square_lupdf`(reals y | reals nu)

The log of the Chi-square density of y given degrees of freedom nu dropping constant additive terms

Available since 2.25

real `chi_square_cdf`(reals y | reals nu)

The Chi-square cumulative distribution function of y given degrees of freedom nu

Available since 2.0

```
real chi_square_lcdf(reals y | reals nu)
```

The log of the Chi-square cumulative distribution function of y given degrees of freedom nu

Available since 2.12

```
real chi_square_lccdf(reals y | reals nu)
```

The log of the complementary Chi-square cumulative distribution function of y given degrees of freedom nu

Available since 2.12

```
R chi_square_rng(reals nu)
```

Generate a Chi-square variate with degrees of freedom nu; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.3. Inverse chi-square distribution

Probability density function

If $\nu \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{InvChiSquare}(y | \nu) = \frac{2^{-\nu/2}}{\Gamma(\nu/2)} y^{-\nu/2-1} \exp\left(-\frac{1}{2} \frac{1}{y}\right).$$

Distribution statement

```
y ~ inv_chi_square(nu)
```

Increment target log probability density with `inv_chi_square_lupdf(y | nu)`.

Available since 2.0

Stan functions

```
real inv_chi_square_lpdf(reals y | reals nu)
```

The log of the inverse Chi-square density of y given degrees of freedom nu

Available since 2.12

```
real inv_chi_square_lupdf(reals y | reals nu)
```

The log of the inverse Chi-square density of y given degrees of freedom nu dropping constant additive terms

Available since 2.25

real `inv_chi_square_cdf`(reals y | reals nu)

The inverse Chi-squared cumulative distribution function of y given degrees of freedom nu

Available since 2.0

real `inv_chi_square_lcdf`(reals y | reals nu)

The log of the inverse Chi-squared cumulative distribution function of y given degrees of freedom nu

Available since 2.12

real `inv_chi_square_lccdf`(reals y | reals nu)

The log of the inverse Chi-squared complementary cumulative distribution function of y given degrees of freedom nu

Available since 2.12

R `inv_chi_square_rng`(reals nu)

Generate an inverse Chi-squared variate with degrees of freedom nu; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.4. Scaled inverse chi-square distribution

Probability density function

If $\nu \in \mathbb{R}^+$ and $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{ScaledInvChiSquare}(y|\nu, \sigma) = \frac{(\nu/2)^{\nu/2}}{\Gamma(\nu/2)} \sigma^\nu y^{-(\nu/2+1)} \exp\left(-\frac{1}{2} \nu \sigma^2 \frac{1}{y}\right).$$

Distribution statement

$y \sim \text{scaled_inv_chi_square}(\text{nu}, \text{sigma})$

Increment target log probability density with `scaled_inv_chi_square_lupdf(y | nu, sigma)`.

Available since 2.0

Stan functions

real `scaled_inv_chi_square_lpdf`(reals y | reals nu, reals sigma)

The log of the scaled inverse Chi-square density of y given degrees of freedom nu and scale sigma

Available since 2.12

real scaled_inv_chi_square_lupdf(reals y | reals nu, reals sigma)

The log of the scaled inverse Chi-square density of y given degrees of freedom nu and scale sigma dropping constant additive terms

Available since 2.25

real scaled_inv_chi_square_cdf(reals y | reals nu, reals sigma)

The scaled inverse Chi-square cumulative distribution function of y given degrees of freedom nu and scale sigma

Available since 2.0

real scaled_inv_chi_square_lcdf(reals y | reals nu, reals sigma)

The log of the scaled inverse Chi-square cumulative distribution function of y given degrees of freedom nu and scale sigma

Available since 2.12

real scaled_inv_chi_square_lccdf(reals y | reals nu, reals sigma)

The log of the scaled inverse Chi-square complementary cumulative distribution function of y given degrees of freedom nu and scale sigma

Available since 2.12

R scaled_inv_chi_square_rng(reals nu, reals sigma)

Generate a scaled inverse Chi-squared variate with degrees of freedom nu and scale sigma; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.5. Exponential distribution

Probability density function

If inverse scale (rate) $\beta \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{Exponential}(y|\beta) = \beta \exp(-\beta y).$$

Distribution statement

$y \sim \text{exponential}(\text{beta})$

Increment target log probability density with `exponential_lupdf(y | beta)`.

Available since 2.0

Stan functions

real `exponential_lpdf`(reals y | reals beta)

The log of the exponential density of y given inverse scale beta

Available since 2.12

real `exponential_lupdf`(reals y | reals beta)

The log of the exponential density of y given inverse scale beta dropping constant additive terms

Available since 2.25

real `exponential_cdf`(reals y | reals beta)

The exponential cumulative distribution function of y given inverse scale beta

Available since 2.0

real `exponential_lcdf`(reals y | reals beta)

The log of the exponential cumulative distribution function of y given inverse scale beta

Available since 2.12

real `exponential_lccdf`(reals y | reals beta)

The log of the exponential complementary cumulative distribution function of y given inverse scale beta

Available since 2.12

R `exponential_rng`(reals beta)

Generate an exponential variate with inverse scale beta; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.6. Gamma distribution

Probability density function

If the shape parameter $\alpha \in \mathbb{R}^+$ and the rate (or inverse scale) parameter $\beta \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{Gamma}(y|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{\alpha-1} \exp(-\beta y).$$

Under the shape and rate formulation of the Gamma distribution, $\mathbb{E}[y] = \alpha/\beta$ and $\text{var}[y] = \alpha/\beta^2$.

Distribution statement

`y ~ gamma(alpha, beta)`

Increment target log probability density with `gamma_lupdf(y | alpha, beta)`.

Available since 2.0

Stan functions

`real gamma_lpdf(reals y | reals alpha, reals beta)`

The log of the gamma density of y given shape alpha and inverse scale beta

Available since 2.12

`real gamma_lupdf(reals y | reals alpha, reals beta)`

The log of the gamma density of y given shape alpha and inverse scale beta dropping constant additive terms

Available since 2.25

`real gamma_cdf(reals y | reals alpha, reals beta)`

The cumulative gamma distribution function of y given shape alpha and inverse scale beta

Available since 2.0

`real gamma_lcdf(reals y | reals alpha, reals beta)`

The log of the cumulative gamma distribution function of y given shape alpha and inverse scale beta

Available since 2.12

`real gamma_lccdf(reals y | reals alpha, reals beta)`

The log of the complementary cumulative gamma distribution function of y given shape alpha and inverse scale beta

Available since 2.12

`R gamma_rng(reals alpha, reals beta)`

Generate a gamma variate with shape alpha and inverse scale beta; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.7. Inverse gamma Distribution

Probability density function

If $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{InvGamma}(y|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} y^{-(\alpha+1)} \exp\left(-\beta \frac{1}{y}\right).$$

Distribution statement

`y ~ inv_gamma(alpha, beta)`

Increment target log probability density with `inv_gamma_lupdf(y | alpha, beta)`.

Available since 2.0

Stan functions

`real inv_gamma_lpdf(reals y | reals alpha, reals beta)`

The log of the inverse gamma density of y given shape alpha and scale beta

Available since 2.12

`real inv_gamma_lupdf(reals y | reals alpha, reals beta)`

The log of the inverse gamma density of y given shape alpha and scale beta dropping constant additive terms

Available since 2.25

`real inv_gamma_cdf(reals y | reals alpha, reals beta)`

The inverse gamma cumulative distribution function of y given shape alpha and scale beta

Available since 2.0

`real inv_gamma_lcdf(reals y | reals alpha, reals beta)`

The log of the inverse gamma cumulative distribution function of y given shape alpha and scale beta

Available since 2.12

`real inv_gamma_lccdf(reals y | reals alpha, reals beta)`

The log of the inverse gamma complementary cumulative distribution function of y given shape alpha and scale beta

Available since 2.12

R `inv_gamma_rng`(reals alpha, reals beta)

Generate an inverse gamma variate with shape alpha and scale beta; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.8. Weibull distribution

Probability density function

If $\alpha \in \mathbb{R}^+$ and $\sigma \in \mathbb{R}^+$, then for $y \in [0, \infty)$,

$$\text{Weibull}(y|\alpha, \sigma) = \frac{\alpha}{\sigma} \left(\frac{y}{\sigma}\right)^{\alpha-1} \exp\left(-\left(\frac{y}{\sigma}\right)^\alpha\right).$$

Note that if $Y \propto \text{Weibull}(\alpha, \sigma)$, then $Y^{-1} \propto \text{Frechet}(\alpha, \sigma^{-1})$.

Distribution statement

`y ~ weibull(alpha, sigma)`

Increment target log probability density with `weibull_lupdf(y | alpha, sigma)`.

Available since 2.0

Stan functions

`real weibull_lpdf(reals y | reals alpha, reals sigma)`

The log of the Weibull density of y given shape alpha and scale sigma

Available since 2.12

`real weibull_lupdf(reals y | reals alpha, reals sigma)`

The log of the Weibull density of y given shape alpha and scale sigma dropping constant additive terms

Available since 2.25

`real weibull_cdf(reals y | reals alpha, reals sigma)`

The Weibull cumulative distribution function of y given shape alpha and scale sigma

Available since 2.0

`real weibull_lcdf(reals y | reals alpha, reals sigma)`

The log of the Weibull cumulative distribution function of y given shape alpha and scale sigma

Available since 2.12

real `weibull_lccdf`(reals y | reals alpha, reals sigma)

The log of the Weibull complementary cumulative distribution function of y given shape alpha and scale sigma

Available since 2.12

R `weibull_rng`(reals alpha, reals sigma)

Generate a weibull variate with shape alpha and scale sigma; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.9. Frechet distribution

Probability density function

If $\alpha \in \mathbb{R}^+$ and $\sigma \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{Frechet}(y|\alpha, \sigma) = \frac{\alpha}{\sigma} \left(\frac{y}{\sigma}\right)^{-\alpha-1} \exp\left(-\left(\frac{y}{\sigma}\right)^{-\alpha}\right).$$

Note that if $Y \propto \text{Frechet}(\alpha, \sigma)$, then $Y^{-1} \propto \text{Weibull}(\alpha, \sigma^{-1})$.

Distribution statement

$y \sim \text{frechet}(\text{alpha}, \text{sigma})$

Increment target log probability density with `frechet_lupdf(y | alpha, sigma)`.

Available since 2.5

Stan functions

real `frechet_lpdf`(reals y | reals alpha, reals sigma)

The log of the Frechet density of y given shape alpha and scale sigma

Available since 2.12

real `frechet_lupdf`(reals y | reals alpha, reals sigma)

The log of the Frechet density of y given shape alpha and scale sigma dropping constant additive terms

Available since 2.25

real `frechet_cdf`(reals y | reals alpha, reals sigma)

The Frechet cumulative distribution function of y given shape alpha and scale sigma

Available since 2.5

real `frechet_lcdf`(reals y | reals alpha, reals sigma)

The log of the Frechet cumulative distribution function of y given shape alpha and scale sigma

Available since 2.12

real `frechet_lccdf`(reals y | reals alpha, reals sigma)

The log of the Frechet complementary cumulative distribution function of y given shape alpha and scale sigma

Available since 2.12

R `frechet_rng`(reals alpha, reals sigma)

Generate a Frechet variate with shape alpha and scale sigma; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.10. Rayleigh distribution

Probability density function

If $\sigma \in \mathbb{R}^+$, then for $y \in [0, \infty)$,

$$\text{Rayleigh}(y|\sigma) = \frac{y}{\sigma^2} \exp(-y^2/2\sigma^2).$$

Distribution statement

`y ~ rayleigh(sigma)`

Increment target log probability density with `rayleigh_lupdf(y | sigma)`.

Available since 2.0

Stan functions

real `rayleigh_lpdf`(reals y | reals sigma)

The log of the Rayleigh density of y given scale sigma

Available since 2.12

real rayleigh_lupdf(reals y | reals sigma)

The log of the Rayleigh density of y given scale sigma dropping constant additive terms

Available since 2.25

real rayleigh_cdf(real y | real sigma)

The Rayleigh cumulative distribution of y given scale sigma

Available since 2.0

real rayleigh_lcdf(real y | real sigma)

The log of the Rayleigh cumulative distribution of y given scale sigma

Available since 2.12

real rayleigh_lccdf(real y | real sigma)

The log of the Rayleigh complementary cumulative distribution of y given scale sigma

Available since 2.12

R rayleigh_rng(reals sigma)

Generate a Rayleigh variate with scale sigma; may only be used in generated quantities block. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

21.11. Log-logistic distribution

Probability density function

If $\alpha, \beta \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$,

$$\text{Log-Logistic}(y|\alpha, \beta) = \frac{\left(\frac{\beta}{\alpha}\right) \left(\frac{y}{\alpha}\right)^{\beta-1}}{\left(1 + \left(\frac{y}{\alpha}\right)^{\beta}\right)^2}.$$

Distribution statement

$y \sim \text{loglogistic}(\text{alpha}, \text{beta})$

Increment target log probability density with unnormalized version of `loglogistic_lpdf(y | alpha, beta)`

Available since 2.29

Stan functions

real `loglogistic_lpdf`(reals y | reals alpha, reals beta)

The log of the log-logistic density of y given scale alpha and shape beta

Available since 2.29

real `loglogistic_cdf`(reals y | reals alpha, reals beta)

The log-logistic cumulative distribution function of y given scale alpha and shape beta

Available since 2.29

R `loglogistic_rng`(reals alpha, reals beta)

Generate a log-logistic variate with scale alpha and shape beta; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.29

22. Positive Lower-Bounded Distributions

The positive lower-bounded probabilities have support on real values above some positive minimum value.

22.1. Pareto distribution

Probability density function

If $y_{\min} \in \mathbb{R}^+$ and $\alpha \in \mathbb{R}^+$, then for $y \in \mathbb{R}^+$ with $y \geq y_{\min}$,

$$\text{Pareto}(y|y_{\min}, \alpha) = \frac{\alpha y_{\min}^\alpha}{y^{\alpha+1}}.$$

Distribution statement

```
y ~ pareto(y_min, alpha)
```

Increment target log probability density with `pareto_lupdf(y | y_min, alpha)`.

Available since 2.0

Stan functions

```
real pareto_lpdf(reals y | reals y_min, reals alpha)
```

The log of the Pareto density of y given positive minimum value `y_min` and shape `alpha`

Available since 2.12

```
real pareto_lupdf(reals y | reals y_min, reals alpha)
```

The log of the Pareto density of y given positive minimum value `y_min` and shape `alpha` dropping constant additive terms

Available since 2.25

```
real pareto_cdf(reals y | reals y_min, reals alpha)
```

The Pareto cumulative distribution function of y given positive minimum value `y_min` and shape `alpha`

Available since 2.0

```
real pareto_lcdf(reals y | reals y_min, reals alpha)
```

The log of the Pareto cumulative distribution function of y given positive minimum value `y_min` and shape `alpha`

Available since 2.12

real `pareto_lccdf`(reals y | reals y_min, reals alpha)

The log of the Pareto complementary cumulative distribution function of y given positive minimum value y_min and shape alpha

Available since 2.12

R `pareto_rng`(reals y_min, reals alpha)

Generate a Pareto variate with positive minimum value y_min and shape alpha; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

22.2. Pareto type 2 distribution

Probability density function

If $\mu \in \mathbb{R}$, $\lambda \in \mathbb{R}^+$, and $\alpha \in \mathbb{R}^+$, then for $y \geq \mu$,

$$\text{Pareto_Type_2}(y|\mu, \lambda, \alpha) = \frac{\alpha}{\lambda} \left(1 + \frac{y - \mu}{\lambda}\right)^{-(\alpha+1)}.$$

Note that the Lomax distribution is a Pareto Type 2 distribution with $\mu = 0$.

Distribution statement

`y ~ pareto_type_2(mu, lambda, alpha)`

Increment target log probability density with `pareto_type_2_lupdf(y | mu, lambda, alpha)`.

Available since 2.5

Stan functions

real `pareto_type_2_lpdf`(reals y | reals mu, reals lambda, reals alpha)

The log of the Pareto Type 2 density of y given location mu, scale lambda, and shape alpha

Available since 2.18

real `pareto_type_2_lupdf`(reals y | reals mu, reals lambda, reals alpha)

The log of the Pareto Type 2 density of y given location mu, scale lambda, and shape alpha dropping constant additive terms

Available since 2.25

```
real pareto_type_2_cdf(reals y | reals mu, reals lambda, reals alpha)
```

The Pareto Type 2 cumulative distribution function of y given location mu, scale lambda, and shape alpha

Available since 2.5

```
real pareto_type_2_lcdf(reals y | reals mu, reals lambda, reals alpha)
```

The log of the Pareto Type 2 cumulative distribution function of y given location mu, scale lambda, and shape alpha

Available since 2.18

```
real pareto_type_2_lccdf(reals y | reals mu, reals lambda, reals alpha)
```

The log of the Pareto Type 2 complementary cumulative distribution function of y given location mu, scale lambda, and shape alpha

Available since 2.18

```
R pareto_type_2_rng(reals mu, reals lambda, reals alpha)
```

Generate a Pareto Type 2 variate with location mu, scale lambda, and shape alpha; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

22.3. Wiener First Passage Time Distribution

For an extended explanation of how to use the `wiener_lpdf` and `wiener_l[c]cdf_unnorm` functions, see Henrich et al. (2024).

Probability density function

If $\alpha \in \mathbb{R}^+$, $\tau \in \mathbb{R}^+$, $\beta \in (0, 1)$, $\delta \in \mathbb{R}$, $s_\delta \in \mathbb{R}^{\geq 0}$, $s_\beta \in [0, 1)$, and $s_\tau \in \mathbb{R}^{\geq 0}$ then for $y > \tau$,

$$\begin{aligned}
\text{Wiener}(y \mid \alpha, \tau, \beta, \delta, s_\delta, s_\beta, s_\tau) = & \\
& \frac{1}{s_\tau} \int_\tau^{\tau+s_\tau} \frac{1}{s_\beta} \int_{\beta-\frac{1}{2}s_\beta}^{\beta+\frac{1}{2}s_\beta} \int_{-\infty}^{\infty} p_3(y - \tau_0 \mid \alpha, \nu, \omega) \\
& \times \frac{1}{\sqrt{2\pi s_\delta^2}} \exp\left(-\frac{(\nu - \delta)^2}{2s_\delta^2}\right) d\nu d\omega d\tau_0 = \\
& \frac{1}{s_\tau} \int_\tau^{\tau+s_\tau} \frac{1}{s_\beta} \int_{\beta-\frac{1}{2}s_\beta}^{\beta+\frac{1}{2}s_\beta} M \times p_3(y - \tau_0 \mid \alpha, \nu, \omega) d\omega d\tau_0,
\end{aligned}$$

where $p()$ denotes the density function, and M and $p_3()$ are defined, by using $t := y - \tau_0$, as

$$\begin{aligned}
M &:= \frac{1}{\sqrt{1+s_\delta^2 t}} \exp\left(\alpha \delta \omega + \frac{\delta^2 t}{2} + \frac{s_\delta^2 \alpha^2 \omega^2 - 2\alpha \delta \omega - \delta^2 t}{2(1+s_\delta^2 t)}\right) \text{ and} \\
p_3(t \mid \alpha, \delta, \beta) &:= \frac{1}{\alpha^2} \exp\left(-\alpha \delta \beta - \frac{\delta^2 t}{2}\right) f\left(\frac{t}{\alpha^2} \mid 0, 1, \beta\right),
\end{aligned}$$

where $f(t^* = \frac{t}{\alpha^2} \mid 0, 1, \beta)$ can be specified in two ways:

$$\begin{aligned}
f_l(t^* \mid 0, 1, \beta) &= \sum_{k=1}^{\infty} k \pi \exp\left(-\frac{k^2 \pi^2 t^*}{2}\right) \sin(k \pi \beta) \text{ and} \\
f_s(t^* \mid 0, 1, \beta) &= \sum_{k=-\infty}^{\infty} \frac{1}{\sqrt{2\pi(t^*)^3}} (\beta + 2k) \exp\left(-\frac{(\beta + 2k)^2}{2t^*}\right).
\end{aligned}$$

Which of these is used in the computations depends on which expression requires the smaller number of components k to guarantee a pre-specified precision

In the case where s_δ, s_β , and s_τ are all 0, this simplifies to one representation that converges fast for small reaction-time values ("small time expansion"):

$$\text{Wiener}(y \mid \alpha, \tau, \beta, \delta) = \frac{\alpha}{(y - \tau)^{3/2}} \exp\left(-\delta \alpha \beta - \frac{\delta^2 (y - \tau)}{2}\right) \sum_{k=-\infty}^{\infty} (2k + \beta) \phi\left(\frac{(2k + \beta)\alpha}{\sqrt{y - \tau}}\right),$$

where $\phi(x)$ denotes the standard normal density function, and one representation that converges fast for large reaction-time values ("large time expansion"):

$$\text{Wiener}(y|\alpha, \tau, \beta, \delta) = \frac{\pi}{\alpha^2} \exp\left(-\delta\alpha\beta - \frac{\delta^2(y - \tau)}{2}\right) \sum_{k=1}^{\infty} k \exp\left(-\frac{k^2\pi^2(y - \tau)}{2\alpha^2}\right) \sin(k\pi\beta)$$

see (Feller 1968), (Navarro and Fuss 2009).

Cumulative distribution function

For the cumulative distribution function (cdf) there also exist two expressions depending on the reaction time.

Let $\alpha, \tau, \beta, \delta, s_{\delta}, s_{\beta}, s_{\tau}$ and y be as above.

The formula for the large-time cdf of decision times (excluding the additive reaction time components summarized in τ for the time being) at the upper boundary is stated as follows:

$$F(y|\alpha, \beta, \delta) = P(\alpha, \beta, \delta) - \exp\left(\delta\alpha(1 - \beta) - \frac{\delta^2 y}{2}\right) F_l(y|\alpha, \beta, \delta), \quad (22.1)$$

where $P(\alpha, \beta, \delta)$ is the probability to hit the upper boundary, defined as

$$P(\alpha, \beta, \delta) = \begin{cases} \frac{1 - \exp(2\delta\alpha\beta)}{\exp(-2\delta\alpha(1 - \beta)) - \exp(2\delta\alpha\beta)}, & \text{for } \delta \neq 0 \\ \beta, & \text{for } \delta = 0, \end{cases} \quad (22.2)$$

and

$$F_l(y|\alpha, \beta, \delta) = \frac{2\pi}{\alpha^2} \sum_{k=1}^{\infty} \frac{k \sin k\pi(1 - \beta)}{\delta^2 + (k\pi)^2/\alpha^2} \exp\left(-\frac{k^2\pi^2 y}{2\alpha^2}\right). \quad (22.3)$$

The formula for the small-time cdf at the upper boundary is stated as follows:

$$F(y|\alpha, \beta, \delta) = \exp\left(\delta\alpha(1 - \beta) - \frac{\delta^2 y}{2}\right) F_s(y|\alpha, \beta, \delta), \quad (22.4)$$

where

$$F_s(y|\alpha, \beta, \delta) = \sum_{k=0}^{\infty} (-1)^k \phi\left(\frac{\alpha(k + \beta_k^*)}{\sqrt{y}}\right) \times \left(R\left(\frac{\alpha(k + \beta_k^*) + \delta y}{\sqrt{y}}\right) + R\left(\frac{\alpha(k + \beta_k^*) - \delta y}{\sqrt{y}}\right)\right), \quad (22.5)$$

where $\beta_k^* = (1 - \beta)$ for k even, $\beta_k^* = \beta$ for k odd, and R is Mill's ratio.

The cdf for the lower boundary is $F(y|\alpha, 1 - \beta, -\delta)$

Distribution statement

`y ~ wiener(alpha, tau, beta, delta)`

Increment target log probability density with `wiener_lupdf(y | alpha, tau, beta, delta)`.

Available since 2.7

`y ~ wiener(alpha, tau, beta, delta, var_delta)` Increment target log probability density with `wiener_lupdf(y | alpha, tau, beta, delta, var_delta)`.

Available since 2.35

`y ~ wiener(alpha, tau, beta, delta, var_delta, var_beta, var_tau)` Increment target log probability density with `wiener_lupdf(y | alpha, tau, beta, delta, var_delta, var_beta, var_tau)`.

Available since 2.35

Stan functions

`real wiener_lpdf(reals y | reals alpha, reals tau, reals beta, reals delta)`

The log of the Wiener first passage time density of y given boundary separation α , non-decision time τ , starting point β , and drift rate δ .

Available since 2.18

`real wiener_lpdf(real y | real alpha, real tau, real beta, real delta, real var_delta)`

The log of the Wiener first passage time density of y given boundary separation α , non-decision time τ , starting point β , drift rate δ , and inter-trial drift rate variability var_δ .

Setting `var_delta` to 0 recovers the 4-parameter signature above.

Available since 2.35

`real wiener_lpdf(real y | real alpha, real tau, real beta, real delta, real var_delta, real var_beta, real var_tau)`

The log of the Wiener first passage time density of y given boundary separation α , non-decision time τ , starting point β , drift rate δ , inter-trial

drift rate variability `var_delta`, inter-trial variability of the starting point (bias) `var_beta`, and inter-trial variability of the non-decision time `var_tau`.

Setting `var_delta`, `var_beta`, and `var_tau` to 0 recovers the 4-parameter signature above.

Available since 2.35

```
real wiener_lupdf(reals y | reals alpha, reals tau, reals beta, reals delta)
```

The log of the Wiener first passage time density of `y` given boundary separation `alpha`, non-decision time `tau`, starting point `beta`, and drift rate `delta`, dropping constant additive terms

Available since 2.25

```
real wiener_lupdf(real y | real alpha, real tau, real beta, real delta, real var_delta)
```

The log of the Wiener first passage time density of `y` given boundary separation `alpha`, non-decision time `tau`, starting point `beta`, drift rate `delta`, and inter-trial drift rate variability `var_delta`, dropping constant additive terms.

Setting `var_delta` to 0 recovers the 4-parameter signature above.

Available since 2.35

```
real wiener_lupdf(real y | real alpha, real tau, real beta, real delta, real var_delta, real var_beta, real var_tau)
```

The log of the Wiener first passage time density of `y` given boundary separation `alpha`, non-decision time `tau`, starting point `beta`, drift rate `delta`, inter-trial drift rate variability `var_delta`, inter-trial variability of the starting point (bias) `var_beta`, and inter-trial variability of the non-decision time `var_tau`, dropping constant additive terms.

Setting `var_delta`, `var_beta`, and `var_tau` to 0 recovers the 4-parameter signature above.

Available since 2.35

Note: The `lcdf` and `lccdf` functions for the `wiener` distribution are conditional and unnormalized, meaning that the cdf does not asymptote at 1, but rather at the probability to hit the upper [boundary](#).

Similarly, the ccdf is defined as the probability to hit the upper boundary less the value of the cdf, as opposed to the more typical $1 - \text{cdf}$.

```
real    wiener_lcdf_unnorm(real y, real alpha, real tau, real beta,  
real delta)
```

The log of the cumulative distribution function (cdf) of the Wiener distribution of y given boundary separation α , non-decision time τ , starting point β , and drift rate δ .

Available since 2.38

```
real    wiener_lccdf_unnorm(real y, real alpha, real tau, real beta,  
real delta)
```

The log of the complementary cumulative distribution function (ccdf) of the Wiener distribution of y given boundary separation α , non-decision time τ , starting point β , and drift rate δ .

Available since 2.38

```
real    wiener_lcdf_unnorm(real y, real alpha, real tau, real beta,  
real delta, real var_delta, real var_beta, real var_tau)
```

The log of the cumulative distribution function (cdf) of the Wiener distribution of y given boundary separation α , non-decision time τ , starting point β , drift rate δ , inter-trial drift rate variability var_delta , inter-trial variability of the starting point (bias) var_beta , and inter-trial variability of the non-decision time var_tau .

Available since 2.38

```
real    wiener_lccdf_unnorm(real y, real alpha, real tau, real beta,  
real delta, real var_delta, real var_beta, real var_tau)
```

The log of the complementary cumulative distribution function (ccdf) of the Wiener distribution of y given boundary separation α , non-decision time τ , starting point β , drift rate δ , inter-trial drift rate variability var_delta , inter-trial variability of the starting point (bias) var_beta , and inter-trial variability of the non-decision time var_tau .

Available since 2.38

Boundaries

Stan returns the first passage time of the accumulation process over the upper boundary only. To get the result for the lower boundary, use

$$\text{Wiener}(y|\alpha, \tau, 1 - \beta, -\delta)$$

For more details, see the appendix of Vandekerckhove and Wabersich (2014).

Vectorization

The 5- and 7-argument forms of the `wiener` distribution functions (listed above as receiving only `real`) are implemented in such a way where they can be fully [vectorized](#), but currently only versions that accept all `real` and all `vector` arguments are exposed by Stan. If there are additional signatures that would prove useful, please request them by [opening an issue](#).

Tolerance tuning

The 5- and 7-argument forms of the `wiener` distribution functions can also accept an additional `data real` argument controlling the required precision of the gradient calculation of the function. If omitted, this defaults to `1e-4` for the density and `1e-8` for the `cdf` functions.

23. Continuous Distributions on $[0, 1]$

The continuous distributions with outcomes in the interval $[0, 1]$ are used to characterize bounded quantities, including probabilities.

23.1. Beta distribution

Probability density function

If $\alpha \in \mathbb{R}^+$ and $\beta \in \mathbb{R}^+$, then for $\theta \in (0, 1)$,

$$\text{Beta}(\theta | \alpha, \beta) = \frac{1}{B(\alpha, \beta)} \theta^{\alpha-1} (1-\theta)^{\beta-1},$$

where the beta function $B()$ is as defined in section [combinatorial functions](#).

Warning: If $\theta = 0$ or $\theta = 1$, then the probability is 0 and the log probability is $-\infty$. Similarly, the distribution requires strictly positive parameters, $\alpha, \beta > 0$.

Distribution statement

```
theta ~ beta(alpha, beta)
```

Increment target log probability density with `beta_lupdf(theta | alpha, beta)`.

Available since 2.0

Stan functions

```
real beta_lpdf(reals theta | reals alpha, reals beta)
```

The log of the beta density of θ in $[0, 1]$ given positive prior successes (plus one) α and prior failures (plus one) β

Available since 2.12

```
real beta_lupdf(reals theta | reals alpha, reals beta)
```

The log of the beta density of θ in $[0, 1]$ given positive prior successes (plus one) α and prior failures (plus one) β dropping constant additive terms

Available since 2.25

```
real beta_cdf(reals theta | reals alpha, reals beta)
```

The beta cumulative distribution function of θ in $[0, 1]$ given positive prior successes (plus one) α and prior failures (plus one) β

Available since 2.0

real `beta_lcdf`(reals theta | reals alpha, reals beta)

The log of the beta cumulative distribution function of theta in $[0, 1]$ given positive prior successes (plus one) alpha and prior failures (plus one) beta

Available since 2.12

real `beta_lccdf`(reals theta | reals alpha, reals beta)

The log of the beta complementary cumulative distribution function of theta in $[0, 1]$ given positive prior successes (plus one) alpha and prior failures (plus one) beta

Available since 2.12

R `beta_rng`(reals alpha, reals beta)

Generate a beta variate with positive prior successes (plus one) alpha and prior failures (plus one) beta; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

23.2. Beta proportion distribution

Probability density function

If $\mu \in (0, 1)$ and $\kappa \in \mathbb{R}^+$, then for $\theta \in (0, 1)$,

$$\text{Beta_Proportion}(\theta | \mu, \kappa) = \frac{1}{B(\mu\kappa, (1 - \mu)\kappa)} \theta^{\mu\kappa - 1} (1 - \theta)^{(1 - \mu)\kappa - 1},$$

where the beta function $B()$ is as defined in section [combinatorial functions](#).

Warning: If $\theta = 0$ or $\theta = 1$, then the probability is 0 and the log probability is $-\infty$. Similarly, the distribution requires $\mu \in (0, 1)$ and strictly positive parameter, $\kappa > 0$.

Distribution statement

`theta ~ beta_proportion(mu, kappa)`

Increment target log probability density with `beta_proportion_lupdf(theta | mu, kappa)`.

Available since 2.19

Stan functions

real `beta_proportion_lpdf`(reals theta | reals mu, reals kappa)

The log of the beta_proportion density of theta in (0,1) given mean mu and precision kappa

Available since 2.19

real `beta_proportion_lupdf`(reals theta | reals mu, reals kappa)

The log of the beta_proportion density of theta in (0,1) given mean mu and precision kappa dropping constant additive terms

Available since 2.25

real `beta_proportion_lcdf`(reals theta | reals mu, reals kappa)

The log of the beta_proportion cumulative distribution function of theta in (0,1) given mean mu and precision kappa

Available since 2.18

real `beta_proportion_lccdf`(reals theta | reals mu, reals kappa)

The log of the beta_proportion complementary cumulative distribution function of theta in (0,1) given mean mu and precision kappa

Available since 2.18

R `beta_proportion_rng`(reals mu, reals kappa)

Generate a beta_proportion variate with mean mu and precision kappa; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

24. Circular Distributions

Circular distributions are defined for finite values y in any interval of length 2π .

24.1. Von Mises distribution

Probability density function

If $\mu \in \mathbb{R}$ and $\kappa \in \mathbb{R}^+$, then for $y \in \mathbb{R}$,

$$\text{VonMises}(y|\mu, \kappa) = \frac{\exp(\kappa \cos(y - \mu))}{2\pi I_0(\kappa)}.$$

In order for this density to properly normalize, y must be restricted to some interval $(c, c + 2\pi)$ of length 2π , because

$$\int_c^{c+2\pi} \text{VonMises}(y|\mu, \kappa) dy = 1.$$

Similarly, if μ is a parameter, it will typically be restricted to the same range as y .

If $\kappa > 0$, a von Mises distribution with its 2π interval of support centered around its location μ will have a single mode at μ ; for example, restricting y to $(-\pi, \pi)$ and taking $\mu = 0$ leads to a single local optimum at the mode μ . If the location μ is not in the center of the support, the density is circularly translated and there will be a second local maximum at the boundary furthest from the mode. Ideally, the parameterization and support will be set up so that the bulk of the probability mass is in a continuous interval around the mean μ .

For $\kappa = 0$, the Von Mises distribution corresponds to the circular uniform distribution with density $1/(2\pi)$ (independently of the values of y or μ).

Distribution statement

`y ~ von_mises(mu, kappa)`

Increment target log probability density with `von_mises_lupdf(y | mu, kappa)`.

Available since 2.0

Stan functions

`real von_mises_lpdf(reals y | reals mu, reals kappa)`

The log of the von mises density of y given location μ and scale κ .

Available since 2.18

real von_mises_lupdf(reals y | reals mu, reals kappa)

The log of the von mises density of y given location mu and scale kappa dropping constant additive terms.

Available since 2.25

real von_mises_cdf(reals y | reals mu, reals kappa)

The von mises cumulative distribution function of y given location mu and scale kappa.

Available since 2.29

real von_mises_lcdf(reals y | reals mu, reals kappa)

The log of the von mises cumulative distribution function of y given location mu and scale kappa.

Available since 2.29

real von_mises_lccdf(reals y | reals mu, reals kappa)

The log of the von mises complementary cumulative distribution function of y given location mu and scale kappa.

Available since 2.29

R von_mises_rng(reals mu, reals kappa)

Generate a Von Mises variate with location mu and scale kappa (i.e. returns values in the interval $[(\mu \bmod 2\pi) - \pi, (\mu \bmod 2\pi) + \pi]$); may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

Numerical stability

Evaluating the Von Mises distribution for $\kappa > 100$ is numerically unstable in the current implementation. Nathanael I. Lichti suggested the following workaround on the Stan users group, based on the fact that as $\kappa \rightarrow \infty$,

$$\text{VonMises}(y|\mu, \kappa) \rightarrow \text{Normal}(\mu, \sqrt{1/\kappa}).$$

The workaround is to replace `y ~ von_mises(mu, kappa)` with

```
if (kappa < 100) {
  y ~ von_mises(mu, kappa);
} else {
```

```
  y ~ normal(mu, sqrt(1 / kappa));  
}
```

25. Bounded Continuous Distributions

The bounded continuous probabilities have support on a finite interval of real numbers.

25.1. Uniform distribution

Probability density function

If $\alpha \in \mathbb{R}$ and $\beta \in (\alpha, \infty)$, then for $y \in [\alpha, \beta]$,

$$\text{Uniform}(y|\alpha, \beta) = \frac{1}{\beta - \alpha}.$$

Distribution statement

`y ~ uniform(alpha, beta)`

Increment target log probability density with `uniform_lupdf(y | alpha, beta)`.

Available since 2.0

Stan functions

`real uniform_lpdf(reals y | reals alpha, reals beta)`

The log of the uniform density of y given lower bound alpha and upper bound beta

Available since 2.12

`real uniform_lupdf(reals y | reals alpha, reals beta)`

The log of the uniform density of y given lower bound alpha and upper bound beta dropping constant additive terms

Available since 2.25

`real uniform_cdf(reals y | reals alpha, reals beta)`

The uniform cumulative distribution function of y given lower bound alpha and upper bound beta

Available since 2.0

`real uniform_lcdf(reals y | reals alpha, reals beta)`

The log of the uniform cumulative distribution function of y given lower bound alpha and upper bound beta

Available since 2.12

real `uniform_lccdf`(reals y | reals alpha, reals beta)

The log of the uniform complementary cumulative distribution function of y given lower bound alpha and upper bound beta

Available since 2.12

R `uniform_rng`(reals alpha, reals beta)

Generate a uniform variate with lower bound alpha and upper bound beta; may only be used in transformed data and generated quantities blocks. For a description of argument and return types, see section [vectorized PRNG functions](#).

Available since 2.18

26. Distributions over Unbounded Vectors

The unbounded vector probability distributions have support on all of \mathbb{R}^K for some fixed K .

26.1. Multivariate normal distribution

Probability density function

If $K \in \mathbb{N}$, $\mu \in \mathbb{R}^K$, and $\Sigma \in \mathbb{R}^{K \times K}$ is symmetric and positive definite, then for $y \in \mathbb{R}^K$,

$$\text{MultiNormal}(y|\mu, \Sigma) = \frac{1}{(2\pi)^{K/2}} \frac{1}{\sqrt{|\Sigma|}} \exp\left(-\frac{1}{2}(y - \mu)^\top \Sigma^{-1} (y - \mu)\right),$$

where $|\Sigma|$ is the absolute determinant of Σ .

Distribution statement

`y ~ multi_normal(mu, Sigma)`

Increment target log probability density with `multi_normal_lupdf(y | mu, Sigma)`.

Available since 2.0

Stan functions

The multivariate normal probability function is overloaded to allow the variate vector y and location vector μ to be vectors or row vectors (or to mix the two types). The density function is also vectorized, so it allows arrays of row vectors or vectors as arguments; see section [vectorized function signatures](#) for a description of vectorization.

`real multi_normal_lpdf(vectors y | vectors mu, matrix Sigma)`

The log of the multivariate normal density of vector(s) y given location vector(s) μ and covariance matrix Σ

Available since 2.12

`real multi_normal_lupdf(vectors y | vectors mu, matrix Sigma)`

The log of the multivariate normal density of vector(s) y given location vector(s) μ and covariance matrix Σ dropping constant additive terms

Available since 2.25

real `multi_normal_lpdf`(vectors y | row_vectors mu, matrix Sigma)

The log of the multivariate normal density of vector(s) y given location row vector(s) mu and covariance matrix Sigma

Available since 2.12

real `multi_normal_lupdf`(vectors y | row_vectors mu, matrix Sigma)

The log of the multivariate normal density of vector(s) y given location row vector(s) mu and covariance matrix Sigma dropping constant additive terms

Available since 2.25

real `multi_normal_lpdf`(row_vectors y | vectors mu, matrix Sigma)

The log of the multivariate normal density of row vector(s) y given location vector(s) mu and covariance matrix Sigma

Available since 2.12

real `multi_normal_lupdf`(row_vectors y | vectors mu, matrix Sigma)

The log of the multivariate normal density of row vector(s) y given location vector(s) mu and covariance matrix Sigma dropping constant additive terms

Available since 2.25

real `multi_normal_lpdf`(row_vectors y | row_vectors mu, matrix Sigma)

The log of the multivariate normal density of row vector(s) y given location row vector(s) mu and covariance matrix Sigma

Available since 2.12

real `multi_normal_lupdf`(row_vectors y | row_vectors mu, matrix Sigma)

The log of the multivariate normal density of row vector(s) y given location row vector(s) mu and covariance matrix Sigma dropping constant additive terms

Available since 2.25

Although there is a direct multi-normal RNG function, if more than one result is required, it's much more efficient to Cholesky factor the covariance matrix and call `multi_normal_cholesky_rng`; see section [multi-variate normal, cholesky parameterization](#).

`vector multi_normal_rng(vector mu, matrix Sigma)`

Generate a multivariate normal variate with location mu and covariance matrix Sigma; may only be used in transformed data and generated quantities blocks

Available since 2.0

`vector multi_normal_rng(row_vector mu, matrix Sigma)`

Generate a multivariate normal variate with location mu and covariance matrix Sigma; may only be used in transformed data and generated quantities blocks

Available since 2.18

`vectors multi_normal_rng(vectors mu, matrix Sigma)`

Generate an array of multivariate normal variates with locations mu and covariance matrix Sigma; may only be used in transformed data and generated quantities blocks

Available since 2.18

`vectors multi_normal_rng(row_vectors mu, matrix Sigma)`

Generate an array of multivariate normal variates with locations mu and covariance matrix Sigma; may only be used in transformed data and generated quantities blocks

Available since 2.18

26.2. Multivariate normal distribution, precision parameterization

Probability density function

If $K \in \mathbb{N}$, $\mu \in \mathbb{R}^K$, and $\Omega \in \mathbb{R}^{K \times K}$ is symmetric and positive definite, then for $y \in \mathbb{R}^K$,

$$\text{MultiNormalPrecision}(y|\mu, \Omega) = \text{MultiNormal}(y|\mu, \Omega^{-1})$$

Distribution statement

`y ~ multi_normal_prec(mu, Omega)`

Increment target log probability density with `multi_normal_prec_lupdf(y | mu, Omega)`.

Available since 2.3

Stan functions

`real multi_normal_prec_lpdf(vectors y | vectors mu, matrix Omega)`

The log of the multivariate normal density of vector(s) y given location vector(s)

mu and positive definite precision matrix Omega

Available since 2.18

real `multi_normal_prec_lupdf`(vectors y | vectors mu, matrix Omega)
 The log of the multivariate normal density of vector(s) y given location vector(s) mu and positive definite precision matrix Omega dropping constant additive terms

Available since 2.25

real `multi_normal_prec_lpdf`(vectors y | row_vectors mu, matrix Omega)

The log of the multivariate normal density of vector(s) y given location row vector(s) mu and positive definite precision matrix Omega

Available since 2.18

real `multi_normal_prec_lupdf`(vectors y | row_vectors mu, matrix Omega)

The log of the multivariate normal density of vector(s) y given location row vector(s) mu and positive definite precision matrix Omega dropping constant additive terms

Available since 2.25

real `multi_normal_prec_lpdf`(row_vectors y | vectors mu, matrix Omega)

The log of the multivariate normal density of row vector(s) y given location vector(s) mu and positive definite precision matrix Omega

Available since 2.18

real `multi_normal_prec_lupdf`(row_vectors y | vectors mu, matrix Omega)

The log of the multivariate normal density of row vector(s) y given location vector(s) mu and positive definite precision matrix Omega dropping constant additive terms

Available since 2.25

real `multi_normal_prec_lpdf`(row_vectors y | row_vectors mu, matrix Omega)

The log of the multivariate normal density of row vector(s) y given location row vector(s) mu and positive definite precision matrix Omega

Available since 2.18

```
real multi_normal_prec_lupdf(row_vectors y | row_vectors mu, matrix Omega)
```

The log of the multivariate normal density of row vector(s) y given location row vector(s) mu and positive definite precision matrix Omega dropping constant additive terms

Available since 2.25

26.3. Multivariate normal distribution, Cholesky parameterization

Probability density function

If $K \in \mathbb{N}$, $\mu \in \mathbb{R}^K$, and $L \in \mathbb{R}^{K \times K}$ is lower triangular and such that LL^\top is positive definite, then for $y \in \mathbb{R}^K$,

$$\text{MultiNormalCholesky}(y|\mu, L) = \text{MultiNormal}(y|\mu, LL^\top).$$

If L is lower triangular and LL^{top} is a $K \times K$ positive definite matrix, then $L_{k,k}$ must be strictly positive for $k \in 1:K$. If an L is provided that is not the Cholesky factor of a positive-definite matrix, the probability functions will raise errors.

Distribution statement

```
y ~ multi_normal_cholesky(mu, L)
```

Increment target log probability density with `multi_normal_cholesky_lupdf(y | mu, L)`.

Available since 2.0

Stan functions

```
real multi_normal_cholesky_lpdf(vectors y | vectors mu, matrix L)
```

The log of the multivariate normal density of vector(s) y given location vector(s) mu and lower-triangular Cholesky factor of the covariance matrix L

Available since 2.18

```
real multi_normal_cholesky_lupdf(vectors y | vectors mu, matrix L)
```

The log of the multivariate normal density of vector(s) y given location vector(s) mu and lower-triangular Cholesky factor of the covariance matrix L dropping constant additive terms

Available since 2.25

```
real multi_normal_cholesky_lpdf(vectors y | row_vectors mu, matrix L)
```

The log of the multivariate normal density of vector(s) y given location row vector(s) μ and lower-triangular Cholesky factor of the covariance matrix L

Available since 2.18

```
real multi_normal_cholesky_lupdf(vectors  $y$  | row_vectors  $\mu$ , matrix  $L$ )
```

The log of the multivariate normal density of vector(s) y given location row vector(s) μ and lower-triangular Cholesky factor of the covariance matrix L dropping constant additive terms

Available since 2.25

```
real multi_normal_cholesky_lpdf(row_vectors  $y$  | vectors  $\mu$ , matrix  $L$ )
```

The log of the multivariate normal density of row vector(s) y given location vector(s) μ and lower-triangular Cholesky factor of the covariance matrix L

Available since 2.18

```
real multi_normal_cholesky_lupdf(row_vectors  $y$  | vectors  $\mu$ , matrix  $L$ )
```

The log of the multivariate normal density of row vector(s) y given location vector(s) μ and lower-triangular Cholesky factor of the covariance matrix L dropping constant additive terms

Available since 2.25

```
real multi_normal_cholesky_lpdf(row_vectors  $y$  | row_vectors  $\mu$ , matrix  $L$ )
```

The log of the multivariate normal density of row vector(s) y given location row vector(s) μ and lower-triangular Cholesky factor of the covariance matrix L

Available since 2.18

```
real multi_normal_cholesky_lupdf(row_vectors  $y$  | row_vectors  $\mu$ , matrix  $L$ )
```

The log of the multivariate normal density of row vector(s) y given location row vector(s) μ and lower-triangular Cholesky factor of the covariance matrix L dropping constant additive terms

Available since 2.25

```
vector multi_normal_cholesky_rng(vector  $\mu$ , matrix  $L$ )
```

Generate a multivariate normal variate with location μ and lower-triangular

Cholesky factor of the covariance matrix L; may only be used in transformed data and generated quantities blocks

Available since 2.3

`vector multi_normal_cholesky_rng(row_vector mu, matrix L)`

Generate a multivariate normal variate with location mu and lower-triangular Cholesky factor of the covariance matrix L; may only be used in transformed data and generated quantities blocks

Available since 2.18

`vectors multi_normal_cholesky_rng(vectors mu, matrix L)`

Generate an array of multivariate normal variates with locations mu and lower-triangular Cholesky factor of the covariance matrix L; may only be used in transformed data and generated quantities blocks

Available since 2.18

`vectors multi_normal_cholesky_rng(row_vectors mu, matrix L)`

Generate an array of multivariate normal variates with locations mu and lower-triangular Cholesky factor of the covariance matrix L; may only be used in transformed data and generated quantities blocks

Available since 2.18

26.4. Multivariate Gaussian process distribution

Probability density function

If $K, N \in \mathbb{N}$, $\Sigma \in \mathbb{R}^{N \times N}$ is symmetric, positive definite kernel matrix and $w \in \mathbb{R}^K$ is a vector of positive inverse scales, then for $y \in \mathbb{R}^{K \times N}$,

$$\text{MultiGP}(y|\Sigma, w) = \prod_{i=1}^K \text{MultiNormal}(y_i|0, w_i^{-1}\Sigma),$$

where y_i is the i th row of y . This is used to efficiently handle Gaussian Processes with multi-variate outputs where only the output dimensions share a kernel function but vary based on their scale. Note that this function does not take into account the mean prediction.

Distribution statement

`y ~ multi_gp(Sigma, w)`

Increment target log probability density with `multi_gp_lupdf(y | Sigma, w)`.

Available since 2.3

Stan functions

`real multi_gp_lpdf(matrix y | matrix Sigma, vector w)`

The log of the multivariate GP density of matrix y given kernel matrix Σ and inverses scales w

Available since 2.12

`real multi_gp_lupdf(matrix y | matrix Sigma, vector w)`

The log of the multivariate GP density of matrix y given kernel matrix Σ and inverses scales w dropping constant additive terms

Available since 2.25

26.5. Multivariate Gaussian process distribution, Cholesky parameterization

Probability density function

If $K, N \in \mathbb{N}$, $L \in \mathbb{R}^{N \times N}$ is lower triangular and such that LL^\top is positive definite kernel matrix (implying $L_{n,n} > 0$ for $n \in 1:N$), and $w \in \mathbb{R}^K$ is a vector of positive inverse scales, then for $y \in \mathbb{R}^{K \times N}$,

$$\text{MultiGPCholesky}(y | L, w) = \prod_{i=1}^K \text{MultiNormal}(y_i | 0, w_i^{-1} LL^\top),$$

where y_i is the i th row of y . This is used to efficiently handle Gaussian Processes with multi-variate outputs where only the output dimensions share a kernel function but vary based on their scale. If the model allows parameterization in terms of Cholesky factor of the kernel matrix, this distribution is also more efficient than `MultiGP()`. Note that this function does not take into account the mean prediction.

Distribution statement

`y ~ multi_gp_cholesky(L, w)`

Increment target log probability density with `multi_gp_cholesky_lupdf(y | L, w)`.

Available since 2.5

Stan functions

`real multi_gp_cholesky_lpdf(matrix y | matrix L, vector w)`

The log of the multivariate GP density of matrix y given lower-triangular Cholesky factor of the kernel matrix L and inverses scales w

Available since 2.12

```
real multi_gp_cholesky_lupdf(matrix y | matrix L, vector w)
```

The log of the multivariate GP density of matrix y given lower-triangular Cholesky factor of the kernel matrix L and inverses scales w dropping constant additive terms

Available since 2.25

26.6. Multivariate Student-t distribution

Probability density function

If $K \in \mathbb{N}$, $\nu \in \mathbb{R}^+$, $\mu \in \mathbb{R}^K$, and $\Sigma \in \mathbb{R}^{K \times K}$ is symmetric and positive definite, then for $y \in \mathbb{R}^K$,

$$\text{MultiStudentT}(y | \nu, \mu, \Sigma) = \frac{1}{\pi^{K/2}} \frac{1}{\nu^{K/2}} \frac{\Gamma((\nu+K)/2)}{\Gamma(\nu/2)} \frac{1}{\sqrt{|\Sigma|}} \left(1 + \frac{1}{\nu} (y - \mu)^\top \Sigma^{-1} (y - \mu)\right)^{-(\nu+K)/2}.$$

Distribution statement

```
y ~ multi_student_t(nu, mu, Sigma)
```

Increment target log probability density with `multi_student_t_lupdf(y | nu, mu, Sigma)`.

Available since 2.0

Stan functions

```
real multi_student_t_lpdf(vectors y | real nu, vectors mu, matrix Sigma)
```

The log of the multivariate Student-*t* density of vector(s) y given degrees of freedom nu, location vector(s) mu, and scale matrix Sigma

Available since 2.18

```
real multi_student_t_lupdf(vectors y | real nu, vectors mu, matrix Sigma)
```

The log of the multivariate Student-*t* density of vector(s) y given degrees of freedom nu, location vector(s) mu, and scale matrix Sigma dropping constant additive terms

Available since 2.25

```
real multi_student_t_lpdf(vectors y | real nu, row_vectors mu, matrix Sigma)
```

The log of the multivariate Student-*t* density of vector(s) *y* given degrees of freedom *nu*, location row vector(s) *mu*, and scale matrix *Sigma*

Available since 2.18

```
real multi_student_t_lupdf(vectors y | real nu, row_vectors mu, matrix Sigma)
```

The log of the multivariate Student-*t* density of vector(s) *y* given degrees of freedom *nu*, location row vector(s) *mu*, and scale matrix *Sigma* dropping constant additive terms

Available since 2.25

```
real multi_student_t_lpdf(row_vectors y | real nu, vectors mu, matrix Sigma)
```

The log of the multivariate Student-*t* density of row vector(s) *y* given degrees of freedom *nu*, location vector(s) *mu*, and scale matrix *Sigma*

Available since 2.18

```
real multi_student_t_lupdf(row_vectors y | real nu, vectors mu, matrix Sigma)
```

The log of the multivariate Student-*t* density of row vector(s) *y* given degrees of freedom *nu*, location vector(s) *mu*, and scale matrix *Sigma* dropping constant additive terms

Available since 2.25

```
real multi_student_t_lpdf(row_vectors y | real nu, row_vectors mu, matrix Sigma)
```

The log of the multivariate Student-*t* density of row vector(s) *y* given degrees of freedom *nu*, location row vector(s) *mu*, and scale matrix *Sigma*

Available since 2.18

```
real multi_student_t_lupdf(row_vectors y | real nu, row_vectors mu, matrix Sigma)
```

The log of the multivariate Student-*t* density of row vector(s) *y* given degrees of freedom *nu*, location row vector(s) *mu*, and scale matrix *Sigma* dropping constant additive terms

Available since 2.25

```
vector multi_student_t_rng(real nu, vector mu, matrix Sigma)
```

Generate a multivariate Student-*t* variate with degrees of freedom *nu*, location

mu, and scale matrix Sigma; may only be used in transformed data and generated quantities blocks

Available since 2.0

`vector multi_student_t_rng(real nu, row_vector mu, matrix Sigma)`

Generate a multivariate Student-*t* variate with degrees of freedom nu, location mu, and scale matrix Sigma; may only be used in transformed data and generated quantities blocks

Available since 2.18

`vectors multi_student_t_rng(real nu, vectors mu, matrix Sigma)`

Generate an array of multivariate Student-*t* variates with degrees of freedom nu, locations mu, and scale matrix Sigma; may only be used in transformed data and generated quantities blocks

Available since 2.18

`vectors multi_student_t_rng(real nu, row_vectors mu, matrix Sigma)`

Generate an array of multivariate Student-*t* variates with degrees of freedom nu, locations mu, and scale matrix Sigma; may only be used in transformed data and generated quantities blocks

Available since 2.18

26.7. Multivariate Student-*t* distribution, Cholesky parameterization

Probability density function

Let $K \in \mathbb{N}$, $\nu \in \mathbb{R}^+$, $\mu \in \mathbb{R}^K$, and L a $K \times K$ lower-triangular matrix with strictly positive, finite diagonal then

$$\text{MultiStudentTCholesky}(y \mid \nu, \mu, L) = \frac{1}{\pi^{K/2}} \frac{1}{\nu^{K/2}} \frac{\Gamma((\nu+K)/2)}{\Gamma(\nu/2)} \frac{1}{|L|} \left(1 + \frac{1}{\nu} (y - \mu)^\top L^{-T} L^{-1} (y - \mu)\right)^{-(\nu+K)/2}.$$

Distribution statement

`y ~ multi_student_t_cholesky(nu, mu, L)`

Increment target log probability density with
`multi_student_t_cholesky_lupdf(y \mid nu, mu, L).`

Available since 2.30

Stan functions

```
real multi_student_t_cholesky_lpdf(vectors y | real nu, vectors mu,
matrix L)
```

The log of the multivariate Student- t density of vector or array of vectors y given degrees of freedom nu, location vector or array of vectors mu, and Cholesky factor of the scale matrix L. For a definition of the arguments compatible with the vectors type, see the [probability vectorization section](#).

Available since 2.30

```
real multi_student_t_cholesky_lupdf(vectors y | real nu, vectors mu,
mu, matrix L)
```

The log of the multivariate Student- t density of vector or vector array y given degrees of freedom nu, location vector or vector array mu, and Cholesky factor of the scale matrix L, dropping constant additive terms. For a definition of arguments compatible with the vectors type, see the [probability vectorization section](#).

Available since 2.30

```
vector multi_student_t_cholesky_rng(real nu, vector mu, matrix L)
```

Generate a multivariate Student- t variate with degrees of freedom nu, location mu, and Cholesky factor of the scale matrix L; may only be used in transformed data and generated quantities blocks.

Available since 2.30

```
array[] vector multi_student_t_cholesky_rng(real nu, array[] vector mu,
mu, matrix L)
```

Generate a multivariate Student- t variate with degrees of freedom nu, location array mu, and Cholesky factor of the scale matrix L; may only be used in transformed data and generated quantities blocks.

Available since 2.30

```
array[] vector multi_student_t_cholesky_rng(real nu, array[]
row_vector mu, matrix L)
```

Generate an array of multivariate Student- t variate with degrees of freedom nu, location array mu, and Cholesky factor of the scale matrix L; may only be used in transformed data and generated quantities blocks.

Available since 2.30

26.8. Gaussian dynamic linear models

A Gaussian Dynamic Linear model is defined as follows, For $t \in 1, \dots, T$,

$$\begin{aligned} y_t &\sim N(F'\theta_t, V) \\ \theta_t &\sim N(G\theta_{t-1}, W) \\ \theta_0 &\sim N(m_0, C_0) \end{aligned}$$

where y is $n \times T$ matrix where rows are variables and columns are observations. These functions calculate the log-density of the observations marginalizing over the latent states ($p(y|F, G, V, W, m_0, C_0)$). This log-density is a system that is calculated using the Kalman Filter. If V is diagonal, then a more efficient algorithm which sequentially processes observations and avoids a matrix inversions can be used (Durbin and Koopman 2001, sec. 6.4).

Distribution statement

`y ~ gaussian_dlm_obs(F, G, V, W, m0, C0)`

Increment target log probability density with `gaussian_dlm_obs_lupdf(y | F, G, V, W, m0, C0)`.

Available since 2.0

Stan functions

The following two functions differ in the type of their V , the first taking a full observation covariance matrix V and the second a vector V representing the diagonal of the observation covariance matrix. The sampling statement defined in the previous section works with either type of observation V .

```
real gaussian_dlm_obs_lpdf(matrix y | matrix F, matrix G, matrix V,
matrix W, vector m0, matrix C0)
```

The log of the density of the Gaussian Dynamic Linear model with observation matrix y in which rows are variables and columns are observations, design matrix F , transition matrix G , observation covariance matrix V , system covariance matrix W , and the initial state is distributed normal with mean m_0 and covariance C_0 .

Available since 2.12

```
real gaussian_dlm_obs_lupdf(matrix y | matrix F, matrix G, matrix V,
matrix W, vector m0, matrix C0)
```

The log of the density of the Gaussian Dynamic Linear model with observation matrix y in which rows are variables and columns are observations, design matrix F , transition matrix G , observation covariance matrix V , system covariance matrix

W , and the initial state is distributed normal with mean $m0$ and covariance $C0$. This function drops constant additive terms.

Available since 2.25

```
real gaussian_dlm_obs_lpdf(matrix y | matrix F, matrix G, vector V,  
matrix W, vector m0, matrix C0)
```

The log of the density of the Gaussian Dynamic Linear model with observation matrix y in which rows are variables and columns are observations, design matrix F , transition matrix G , observation covariance matrix with diagonal V , system covariance matrix W , and the initial state is distributed normal with mean $m0$ and covariance $C0$.

Available since 2.12

```
real gaussian_dlm_obs_lupdf(matrix y | matrix F, matrix G, vector V,  
matrix W, vector m0, matrix C0)
```

The log of the density of the Gaussian Dynamic Linear model with observation matrix y in which rows are variables and columns are observations, design matrix F , transition matrix G , observation covariance matrix with diagonal V , system covariance matrix W , and the initial state is distributed normal with mean $m0$ and covariance $C0$. This function drops constant additive terms.

Available since 2.25

27. Simplex Distributions

The simplex probabilities have support on the unit K -simplex for a specified K . A K -dimensional vector θ is a unit K -simplex if $\theta_k \geq 0$ for $k \in \{1, \dots, K\}$ and $\sum_{k=1}^K \theta_k = 1$.

27.1. Dirichlet distribution

Probability density function

If $K \in \mathbb{N}$ and $\alpha \in (\mathbb{R}^+)^K$, then for $\theta \in K$ -simplex,

$$\text{Dirichlet}(\theta|\alpha) = \frac{\Gamma\left(\sum_{k=1}^K \alpha_k\right)}{\prod_{k=1}^K \Gamma(\alpha_k)} \prod_{k=1}^K \theta_k^{\alpha_k-1}$$

Warning: If any of the components of θ satisfies $\theta_i = 0$ or $\theta_i = 1$, then the probability is 0 and the log probability is $-\infty$. Similarly, the distribution requires strictly positive parameters, with $\alpha_i > 0$ for each i .

Meaning of Dirichlet parameters

A symmetric Dirichlet prior is $[\alpha, \dots, \alpha]^\top$. To code this in Stan,

```
data {
  int<lower=1> K;
  real<lower=0> alpha;
}
generated quantities {
  vector[K] theta = dirichlet_rng(rep_vector(alpha, K));
}
```

Taking $K = 10$, here are the first five draws for $\alpha = 1$. For $\alpha = 1$, the distribution is uniform over simplexes.

- 1) 0.17 0.05 0.07 0.17 0.03 0.13 0.03 0.03 0.27 0.05
- 2) 0.08 0.02 0.12 0.07 0.52 0.01 0.07 0.04 0.01 0.06
- 3) 0.02 0.03 0.22 0.29 0.17 0.10 0.09 0.00 0.05 0.03
- 4) 0.04 0.03 0.21 0.13 0.04 0.01 0.10 0.04 0.22 0.18
- 5) 0.11 0.22 0.02 0.01 0.06 0.18 0.33 0.04 0.01 0.01

That does not mean it's uniform over the marginal probabilities of each element. As the size of the simplex grows, the marginal draws become more and more concentrated below (not around) 1/K. When one component of the simplex is large, the others must all be relatively small to compensate. For example, in a uniform distribution on 10-simplexes, the probability that a component is greater than the mean of 1/10 is only 39%. Most of the posterior marginal probability mass for each component is in the interval (0, 0.1).

When the α value is small, the draws gravitate to the corners of the simplex. Here are the first five draws for $\alpha = 0.001$.

```
1) 3e-203 0e+00 2e-298 9e-106 1e+000 0e+000 0e+000 1e-047 0e+000 4e-279
2) 1e+000 0e+000 5e-279 2e-014 1e-275 0e+000 3e-285 9e-147 0e+000 0e+000
3) 1e-308 0e+000 1e-213 0e+000 0e+000 8e-75 0e+000 1e+000 4e-58 7e-112
4) 6e-166 5e-65 3e-068 3e-147 0e+000 1e+000 3e-249 0e+000 0e+000 0e+000
5) 2e-091 0e+000 0e+000 0e+000 1e-060 0e+000 4e-312 1e+000 0e+000 0e+000
```

Each row denotes a draw. Each draw has a single value that rounds to one and other values that are very close to zero or rounded down to zero.

As α increases, the draws become increasingly uniform. For $\alpha = 1000$,

```
1) 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10
2) 0.10 0.10 0.09 0.10 0.10 0.10 0.11 0.10 0.10 0.10
3) 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10
4) 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10
5) 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10 0.10
```

Distribution statement

`theta ~ dirichlet(alpha)`

Increment target log probability density with `dirichlet_lupdf(theta | alpha)`.

Available since 2.0

Stan functions

The Dirichlet probability functions are overloaded to allow the simplex θ and prior counts (plus one) α to be vectors or row vectors (or to mix the two types). The density functions are also vectorized, so they allow arrays of row vectors or vectors as arguments; see section [vectorized function signatures](#) for a description of vectorization.

`real dirichlet_lpdf(vectors theta | vectors alpha)`

The log of the Dirichlet density for simplex(es) θ given prior counts (plus one) α

Available since 2.12, vectorized in 2.21

```
real dirichlet_lupdf(vectors theta | vectors alpha)
```

The log of the Dirichlet density for simplex(es) theta given prior counts (plus one) alpha dropping constant additive terms

Available since 2.25

```
vector dirichlet_rng(vector alpha)
```

Generate a Dirichlet variate with prior counts (plus one) alpha; may only be used in transformed data and generated quantities blocks

Available since 2.0

28. Correlation Matrix Distributions

The correlation matrix distributions have support on the (Cholesky factors of) correlation matrices. A Cholesky factor L for a $K \times K$ correlation matrix Σ of dimension K has rows of unit length so that the diagonal of LL^\top is the unit K -vector. Even though models are usually conceptualized in terms of correlation matrices, it is better to operationalize them in terms of their Cholesky factors. If you are interested in the posterior distribution of the correlations, you can recover them in the generated quantities block via

```
generated quantities {
  corr_matrix[K] Sigma;
  Sigma = multiply_lower_tri_self_transpose(L);
}
```

28.1. LKJ correlation distribution

Probability density function

For $\eta > 0$, if Σ a positive-definite, symmetric matrix with unit diagonal (i.e., a correlation matrix), then

$$\text{LkjCorr}(\Sigma|\eta) \propto \det(\Sigma)^{(\eta-1)}.$$

The expectation is the identity matrix for any positive value of the shape parameter η , which can be interpreted like the shape parameter of a symmetric beta distribution:

- if $\eta = 1$, then the density is uniform over correlation matrices of order K ;
- if $\eta > 1$, the identity matrix is the modal correlation matrix, with a sharper peak in the density at the identity matrix for larger η ; and
- for $0 < \eta < 1$, the density has a trough at the identity matrix.
- if η were an unknown parameter, the Jeffreys prior is proportional to $\sqrt{2 \sum_{k=1}^{K-1} \left(\psi_1 \left(\eta + \frac{K-k-1}{2} \right) - 2\psi_1(2\eta + K - k - 1) \right)}$, where $\psi_1()$ is the trigamma function

See (Lewandowski, Kurowicka, and Joe 2009) for definitions. However, it is much

better computationally to work directly with the Cholesky factor of Σ , so this distribution should never be explicitly used in practice.

Distribution statement

`y ~ lkj_corr(eta)`

Increment target log probability density with `lkj_corr_lupdf(y | eta)`.

Available since 2.3

Stan functions

`real lkj_corr_lpdf(matrix y | real eta)`

The log of the LKJ density for the correlation matrix `y` given nonnegative shape `eta`. `lkj_corr_cholesky_lpdf` is faster, more numerically stable, uses less memory, and should be preferred to this.

Available since 2.12

`real lkj_corr_lupdf(matrix y | real eta)`

The log of the LKJ density for the correlation matrix `y` given nonnegative shape `eta` dropping constant additive terms. `lkj_corr_cholesky_lupdf` is faster, more numerically stable, uses less memory, and should be preferred to this.

Available since 2.25

`matrix lkj_corr_rng(int K, real eta)`

Generate a LKJ random correlation matrix of order `K` with shape `eta`; may only be used in transformed data and generated quantities blocks

Available since 2.0

28.2. Cholesky LKJ correlation distribution

Stan provides an implicit parameterization of the LKJ correlation matrix density in terms of its Cholesky factor, which you should use rather than the explicit parameterization in the previous section. For example, if `L` is a Cholesky factor of a correlation matrix, then

`L ~ lkj_corr_cholesky(2.0); # implies L * L' ~ lkj_corr(2.0);`

Because Stan requires models to have support on all valid constrained parameters, `L` will almost always¹ be a parameter declared with the type of a Cholesky factor for a correlation matrix; for example,

¹It is possible to build up a valid `L` within Stan, but that would then require Jacobian adjustments to imply the intended posterior.

```
parameters { cholesky_factor_corr[K] L; # rather than corr_matrix[K]
```

Probability density function

For $\eta > 0$, if L is a $K \times K$ lower-triangular Cholesky factor of a symmetric positive-definite matrix with unit diagonal (i.e., a correlation matrix), then

$$\text{LkjCholesky}(L|\eta) \propto |J| \det(LL^\top)^{(\eta-1)} = \prod_{k=2}^K L_{kk}^{K-k+2\eta-2}.$$

See the previous section for details on interpreting the shape parameter η . Note that even if $\eta = 1$, it is still essential to evaluate the density function because the density of L is not constant, regardless of the value of η , even though the density of LL^\top is constant iff $\eta = 1$.

A lower triangular L is a Cholesky factor for a correlation matrix if and only if $L_{k,k} > 0$ for $k \in 1:K$ and each row L_k has unit Euclidean length.

Distribution statement

$L \sim \text{lkj_corr_cholesky}(\text{eta})$

Increment target log probability density with `lkj_corr_cholesky_lupdf(L | eta)`.

Available since 2.4

Stan functions

`real lkj_corr_cholesky_lpdf(matrix L | real eta)`

The log of the LKJ density for the lower-triangular Cholesky factor L of a correlation matrix given shape η

Available since 2.12

`real lkj_corr_cholesky_lupdf(matrix L | real eta)`

The log of the LKJ density for the lower-triangular Cholesky factor L of a correlation matrix given shape η dropping constant additive terms

Available since 2.25

`matrix lkj_corr_cholesky_rng(int K, real eta)`

Generate a random Cholesky factor of a correlation matrix of order K that is distributed LKJ with shape η ; may only be used in transformed data and generated quantities blocks

Available since 2.4

29. Covariance Matrix Distributions

The covariance matrix distributions have support on symmetric, positive-definite $K \times K$ matrices or their Cholesky factors (square, lower triangular matrices with positive diagonal elements).

29.1. Wishart distribution

Probability density function

If $K \in \mathbb{N}$, $\nu \in (K - 1, \infty)$, and $S \in \mathbb{R}^{K \times K}$ is symmetric and positive definite, then for symmetric and positive-definite $W \in \mathbb{R}^{K \times K}$,

$$\text{Wishart}(W \mid \nu, S) = \frac{1}{2^{\nu K/2}} \frac{1}{\Gamma_K(\frac{\nu}{2})} |S|^{-\nu/2} |W|^{(\nu-K-1)/2} \exp\left(-\frac{1}{2} \text{tr}(S^{-1}W)\right),$$

where $\text{tr}()$ is the matrix trace function, and $\Gamma_K()$ is the multivariate Gamma function,

$$\Gamma_K(x) = \frac{1}{\pi^{K(K-1)/4}} \prod_{k=1}^K \Gamma\left(x + \frac{1-k}{2}\right).$$

Distribution statement

`W ~ wishart(nu, Sigma)`

Increment target log probability density with `wishart_lupdf(W | nu, Sigma)`.

Available since 2.0

Stan functions

`real wishart_lpdf(matrix W | real nu, matrix Sigma)`

Return the log of the Wishart density for symmetric and positive-definite matrix W given degrees of freedom ν and symmetric and positive-definite scale matrix Σ .

Available since 2.12

`real wishart_lupdf(matrix W | real nu, matrix Sigma)`

Return the log of the Wishart density for symmetric and positive-definite matrix W given degrees of freedom ν and symmetric and positive-definite scale matrix Σ dropping constant additive terms.

Available since 2.25

matrix wishart_rng(real nu, matrix Sigma)

Generate a Wishart variate with degrees of freedom `nu` and symmetric and positive-definite scale matrix `Sigma`; may only be used in transformed data and generated quantities blocks.

Available since 2.0

29.2. Wishart distribution, Cholesky Parameterization

The Cholesky parameterization of the Wishart distribution uses a Cholesky factor for both the variate and the parameter. If S and W are positive definite matrices with Cholesky factors L_S and L_W (i.e., $S = L_S L_S^\top$ and $W = L_W L_W^\top$), then the Cholesky parameterization is defined so that

$$L_W \sim \text{WishartCholesky}(\nu, L_S)$$

if and only if

$$W \sim \text{Wishart}(\nu, S).$$

Probability density function

If $K \in \mathbb{N}$, $\nu \in (K - 1, \infty)$, and $L_S, L_W \in \mathbb{R}^{K \times K}$ are lower triangular matrixes with positive diagonal elements, then the Cholesky parameterized Wishart density is

$$\text{WishartCholesky}(L_W \mid \nu, L_S) = \text{Wishart}(L_W L_W^\top \mid \nu, L_S L_S^\top) \left| J_{f^{-1}} \right|,$$

where $J_{f^{-1}}$ is the Jacobian of the (inverse) transform of the variate, $f^{-1}(L_W) = L_W L_W^\top$. The log absolute determinant is

$$\log \left| J_{f^{-1}} \right| = K \log(2) + \sum_{k=1}^K (K - k + 1) \log(L_W)_{k,k}.$$

The probability functions will raise errors if $\nu \leq K - 1$ or if L_S and L_W are not Cholesky factors (square, lower-triangular matrices with positive diagonal elements) of the same size.

Stan functions

real wishart_cholesky_lpdf(matrix L_W | real nu, matrix L_S)

Return the log of the Wishart density for lower-triangular Cholesky factor `L_W` given degrees of freedom `nu` and lower-triangular Cholesky factor of the scale matrix `L_S`.

Available since 2.30

real `wishart_cholesky_lupdf`(matrix L_W | real nu, matrix L_S)

Return the log of the Wishart density for lower-triangular Cholesky factor of L_W given degrees of freedom nu and lower-triangular Cholesky factor of the scale matrix L_S dropping constant additive terms.

Available since 2.30

matrix `wishart_cholesky_rng`(real nu, matrix L_S)

Generate the Cholesky factor of a Wishart variate with degrees of freedom nu and lower-triangular Cholesky factor of the scale matrix L_S; may only be used in transformed data and generated quantities blocks

Available since 2.30

29.3. Inverse Wishart distribution

Probability density function

If $K \in \mathbb{N}$, $\nu \in (K - 1, \infty)$, and $S \in \mathbb{R}^{K \times K}$ is symmetric and positive definite, then for symmetric and positive-definite $W \in \mathbb{R}^{K \times K}$,

$$\text{InvWishart}(W | \nu, S) = \frac{1}{2^{\nu K/2}} \frac{1}{\Gamma_K(\frac{\nu}{2})} |S|^{\nu/2} |W|^{-(\nu+K+1)/2} \exp\left(-\frac{1}{2} \text{tr}(SW^{-1})\right).$$

Distribution statement

$W \sim \text{inv_wishart}(\nu, \Sigma)$

Increment target log probability density with `inv_wishart_lupdf(W | nu, Sigma)`.

Available since 2.0

Stan functions

real `inv_wishart_lpdf`(matrix W | real nu, matrix Sigma)

Return the log of the inverse Wishart density for symmetric and positive-definite matrix W given degrees of freedom nu and symmetric and positive-definite scale matrix Sigma.

Available since 2.12

real `inv_wishart_lupdf`(matrix W | real nu, matrix Sigma)

Return the log of the inverse Wishart density for symmetric and positive-definite

matrix W given degrees of freedom nu and symmetric and positive-definite scale matrix $Sigma$ dropping constant additive terms.

Available since 2.25

matrix inv_wishart_rng(real nu, matrix Sigma)

Generate an inverse Wishart variate with degrees of freedom nu and symmetric and positive-definite scale matrix $Sigma$; may only be used in transformed data and generated quantities blocks.

Available since 2.0

29.4. Inverse Wishart distribution, Cholesky Parameterization

The Cholesky parameterization of the inverse Wishart distribution uses a Cholesky factor for both the variate and the parameter. If S and W are positive definite matrices with Cholesky factors L_S and L_W (i.e., $S = L_S L_S^\top$ and $W = L_W L_W^\top$), then the Cholesky parameterization is defined so that

$$L_W \sim \text{InvWishartCholesky}(\nu, L_S)$$

if and only if

$$W \sim \text{InvWishart}(\nu, S).$$

Probability density function

If $K \in \mathbb{N}$, $\nu \in (K - 1, \infty)$, and $L_S, L_W \in \mathbb{R}^{K \times K}$ are lower triangular matrixes with positive diagonal elements, then the Cholesky parameterized inverse Wishart density is

$$\text{InvWishartCholesky}(L_W \mid \nu, L_S) = \text{InvWishart}(L_W L_W^\top \mid \nu, L_S L_S^\top) \left| J_{f^{-1}} \right|,$$

where $J_{f^{-1}}$ is the Jacobian of the (inverse) transform of the variate, $f^{-1}(L_W) = L_W L_W^\top$. The log absolute determinant is

$$\log \left| J_{f^{-1}} \right| = K \log(2) + \sum_{k=1}^K (K - k + 1) \log (L_W)_{k,k}.$$

The probability functions will raise errors if $\nu \leq K - 1$ or if L_S and L_W are not Cholesky factors (square, lower-triangular matrices with positive diagonal elements) of the same size.

Stan functions

real `inv_wishart_cholesky_lpdf`(matrix L_W | real ν , matrix L_S)

Return the log of the inverse Wishart density for lower-triangular Cholesky factor L_W given degrees of freedom ν and lower-triangular Cholesky factor of the scale matrix L_S .

Available since 2.30

real `inv_wishart_cholesky_lupdf`(matrix L_W | real ν , matrix L_S)

Return the log of the inverse Wishart density for lower-triangular Cholesky factor of L_W given degrees of freedom ν and lower-triangular Cholesky factor of the scale matrix L_S dropping constant additive terms.

Available since 2.30

matrix `inv_wishart_cholesky_rng`(real ν , matrix L_S)

Generate the Cholesky factor of an inverse Wishart variate with degrees of freedom ν and lower-triangular Cholesky factor of the scale matrix L_S ; may only be used in transformed data and generated quantities blocks.

Available since 2.30

Part IV

Additional Distributions

30. Hidden Markov Models

An elementary first-order Hidden Markov model is a probabilistic model over N observations, y_n , and N hidden states, x_n , which can be fully defined by the conditional distributions $p(y_n \mid x_n, \phi)$ and $p(x_n \mid x_{n-1}, \phi)$. Here we make the dependency on additional model parameters, ϕ , explicit. When x is continuous, the user can explicitly encode these distributions in Stan and use Markov chain Monte Carlo to integrate x out.

When each state x takes a value over a discrete and finite set, say $\{1, 2, \dots, K\}$, we can take advantage of the dependency structure to marginalize x and compute $p(y \mid \phi)$. We start by defining the conditional observational distribution, stored in a $K \times N$ matrix ω with

$$\omega_{kn} = p(y_n \mid x_n = k, \phi).$$

Next, we introduce the $K \times K$ transition matrix, Γ , with

$$\Gamma_{ij} = p(x_n = j \mid x_{n-1} = i, \phi).$$

Each row defines a probability distribution and must therefore be a simplex (i.e. its components must add to 1). Currently, Stan only supports stationary transitions where a single transition matrix is used for all transitions. Finally we define the initial state K -vector ρ , with

$$\rho_k = p(x_0 = k \mid \phi).$$

The Stan functions that support this type of model are special in that the user does not explicitly pass y and ϕ as arguments. Instead, the user passes $\log \omega$, Γ , and ρ , which in turn depend on y and ϕ .

30.1. Stan functions

```
real hmm_marginal(matrix log_omega, matrix Gamma, vector rho)  
Returns the log probability density of  $y$ , with  $x_n$  integrated out at each iteration.
```

Available since 2.24

The arguments represent (1) the log density of each output, (2) the transition matrix, and (3) the initial state vector.

- \log_omega : $\log \omega_{kn} = \log p(y_n \mid x_n = k, \phi)$, log density of each output,

- *Gamma*: $\Gamma_{ij} = p(x_n = j | x_{n-1} = i, \phi)$, the transition matrix,
- *rho*: $\rho_k = p(x_0 = k | \phi)$, the initial state probability.

```
array[] int hmm_latent_rng(matrix log_omega, matrix Gamma, vector rho)
```

Returns a length N array of integers over $\{1, \dots, K\}$, sampled from the joint posterior distribution of the hidden states, $p(x | \phi, y)$. May be only used in transformed data and generated quantities.

Available since 2.24

```
matrix hmm_hidden_state_prob(matrix log_omega, matrix Gamma, vector rho)
```

Returns the matrix of marginal posterior probabilities of each hidden state value. This will be a $K \times N$ matrix. The n^{th} column is a simplex of probabilities for the n^{th} variable. Moreover, let A be the output. Then $A_{ij} = p(x_j = i | \phi, y)$. This function may only be used in transformed data and generated quantities.

Available since 2.24

Part V

Appendix

31. Mathematical Functions

This appendix provides the definition of several mathematical functions used throughout the manual.

31.1. Beta

The beta function, $B(a, b)$, computes the normalizing constant for the beta distribution, and is defined for $a > 0$ and $b > 0$ by

$$B(a, b) = \int_0^1 u^{a-1} (1-u)^{b-1} du = \frac{\Gamma(a) \Gamma(b)}{\Gamma(a+b)},$$

where $\Gamma(x)$ is the [Gamma function](#).

31.2. Incomplete beta

The incomplete beta function, $B(x; a, b)$, is defined for $x \in [0, 1]$ and $a, b \geq 0$ such that $a + b \neq 0$ by

$$B(x; a, b) = \int_0^x u^{a-1} (1-u)^{b-1} du,$$

where $B(a, b)$ is the beta function defined in [appendix](#). If $x = 1$, the incomplete beta function reduces to the beta function, $B(1; a, b) = B(a, b)$.

The regularized incomplete beta function divides the incomplete beta function by the beta function,

$$I_x(a, b) = \frac{B(x; a, b)}{B(a, b)}.$$

31.3. Gamma

The gamma function, $\Gamma(x)$, is the generalization of the factorial function to continuous variables, defined so that for positive integers n ,

$$\Gamma(n+1) = n!$$

Generalizing to all positive numbers and non-integer negative numbers,

$$\Gamma(x) = \int_0^\infty u^{x-1} \exp(-u) du.$$

31.4. Digamma

The digamma function Ψ is the derivative of the $\log \Gamma$ function,

$$\Psi(u) = \frac{d}{du} \log \Gamma(u) = \frac{1}{\Gamma(u)} \frac{d}{du} \Gamma(u).$$

References

Bailey, David H., Karthik Jeyabalan, and Xiaoye S. Li. 2005. "A Comparison of Three High-Precision Quadrature Schemes." *Experiment. Math.* 14 (3): 317–29. <https://projecteuclid.org:443/euclid.em/1128371757>.

Bowling, Shannon R., Mohammad T. Khasawneh, Sittichai Kaewkuekool, and Byung Rae Cho. 2009. "A Logistic Approximation to the Cumulative Normal Distribution." *Journal of Industrial Engineering and Management* 2 (1): 114–27.

Ding, Peng, and Joseph K. Blitzstein. 2018. "On the Gaussian Mixture Representation of the Laplace Distribution." *The American Statistician* 72 (2): 172–74. <https://doi.org/10.1080/00031305.2017.1291448>.

Durbin, J., and S. J. Koopman. 2001. *Time Series Analysis by State Space Methods*. New York: Oxford University Press.

Feller, William. 1968. *An Introduction to Probability Theory and Its Applications*. Vol. 1. 3. Wiley, New York.

Gaebler, Johann D. 2021. "Autodiff for Implicit Functions in Stan." <https://www.jg.aeb.com/2021/09/13/implicit-autodiff.html#fn:7>.

Gelman, Andrew, J. B. Carlin, Hal S. Stern, David B. Dunson, Aki Vehtari, and Donald B. Rubin. 2013. *Bayesian Data Analysis*. Third Edition. London: Chapman & Hall / CRC Press.

Golub, G. H., and V. Pereyra. 1973. "The Differentiation of Pseudo-Inverses and Nonlinear Least Squares Problems Whose Variables Separate." *SIAM Journal on Numerical Analysis* 10 (2): 413–32. <https://doi.org/10.1137/0710036>.

Guennebaud, Gaël, Benoît Jacob, et al. 2010. "Eigen V3." <http://eigen.tuxfamily.org>.

Henrich, Franziska, Raphael Hartmann, Valentin Pratz, Andreas Voss, and Karl Christoph Klauer. 2024. "The Seven-Parameter Diffusion Model: An Implementation in Stan for Bayesian Analyses." *Behavior Research Methods* 56 (4): 3102–16. <https://doi.org/10.3758/s13428-023-02179-1>.

Hindmarsh, Alan C, Peter N Brown, Keith E Grant, Steven L Lee, Radu Serban, Dan E Shumaker, and Carol S Woodward. 2005. "SUNDIALS: Suite of Nonlinear and Differential/Algebraic Equation Solvers." *ACM Transactions on Mathematical Software (TOMS)* 31 (3): 363–96.

Jorge J. More, Kenneth E. Hillstrom, Burton S. Garbow. 1980. *User Guide for MINPACK-1*. 9700 South Cass Avenue, Argonne, Illinois 60439: Argonne National Laboratory.

Lewandowski, Daniel, Dorota Kurowicka, and Harry Joe. 2009. "Generating Random Correlation Matrices Based on Vines and Extended Onion Method." *Journal of Multivariate Analysis* 100: 1989–2001.

Margossian, Charles C, and Michael Betancourt. 2022. "Efficient Automatic Differentiation of Implicit Functions." *Preprint. arXiv:2112.14217*.

Mori, Masatake. 1978. "An IMT-Type Double Exponential Formula for Numerical Integration." *Publications of the Research Institute for Mathematical Sciences* 14 (3): 713–29. <https://doi.org/10.2977/prims/1195188835>.

Navarro, Danielle J, and Ian G Fuss. 2009. "Fast and Accurate Calculations for First-Passage Times in Wiener Diffusion Models." *Journal of Mathematical Psychology* 53 (4): 222–30.

Powell, Michael J. D. 1970. "A Hybrid Method for Nonlinear Equations." In *Numerical Methods for Nonlinear Algebraic Equations*, edited by P. Rabinowitz. Gordon; Breach.

Serban, Radu, Cosmin Petra, Alan C. Hindmarsh, Cody J. Balos, David J. Gardner, Daniel R. Reynolds, and Carol S. Woodward. 2021. "User Documentation for IDAS V5.0.0." Lawrence Livermore National Laboratory.

Takahasi, Hidetosi, and Masatake Mori. 1974. "Double Exponential Formulas for Numerical Integration." *Publications of the Research Institute for Mathematical Sciences* 9 (3): 721–41. <https://doi.org/10.2977/prims/1195192451>.

Tanaka, Ken'ichiro, Masaaki Sugihara, Kazuo Murota, and Masatake Mori. 2009. "Function Classes for Double Exponential Integration Formulas." *Numerische Mathematik* 111 (4): 631–55. <https://doi.org/10.1007/s00211-008-0195-1>.

Vandekerckhove, Joachim, and Dominik Wabersich. 2014. "The RWiener Package: An R Package Providing Distribution Functions for the Wiener Diffusion Model." *The R Journal* 6/1. <http://journal.r-project.org/archive/2014-1/vandekerckhove-wabersich.pdf>.

Wichura, Michael J. 1988. "Algorithm AS 241: The Percentage Points of the Normal Distribution." *Journal of the Royal Statistical Society. Series C (Applied Statistics)* 37 (3): 477–84. <http://www.jstor.org/stable/2347330>.

Index

abs
(T x): T, 24
(complex z): real, 47

acos
(T x): R, 28
(complex z): complex, 50

acosh
(T x): R, 29
(complex z): complex, 51

add_diag
(complex_matrix m, complex d):
 complex_matrix, 119
(complex_matrix m, complex_row_vector d):
 complex_matrix, 119
(complex_matrix m, complex_vector d):
 complex_matrix, 119
(matrix m, real d): matrix, 79
(matrix m, row_vector d): matrix, 79
(matrix m, vector d): matrix, 79

algebra_solver
(function algebra_system, vector
 y_guess, vector theta, data
 array[] real x_r, array[] int
 x_i, data real rel_tol, data
 real f_tol, int max_steps):
 vector, 173

algebra_solver_newton
(function algebra_system, vector
 y_guess, vector theta, data
 array[] real x_r, array[] int
 x_i): vector, 173
(function algebra_system, vector
 y_guess, vector theta, data
 array[] real x_r, array[] int
 x_i, data real rel_tol, data
 real f_tol, int max_steps):
 vector, 173

append_array
(T x, T y): T, 60

append_col
(complex x, complex_row_vector y):
 complex_row_vector, 121
(complex_matrix x, complex_matrix
 y): complex_matrix, 121
(complex_matrix x, complex_vector
 y): complex_matrix, 121

append_row
(complex x, complex_vector y):
 complex_vector, 122
(complex_matrix x, complex_matrix
 y): complex_matrix, 122
(complex_matrix x, complex_row_vector y):
 complex_matrix, 122
(complex_row_vector x, complex_matrix
 y): complex_matrix, 122
(complex_row_vector x, complex_row_vector y):
 complex_matrix, 122
(complex_vector x, complex y):
 complex_vector, 122
(complex_vector x, complex_vector
 y): complex_vector, 122
(matrix x, matrix y): matrix, 84
(matrix x, vector y): matrix, 84
(real x, row_vector y): row_vector,
 85
(row_vector x, real y): row_vector,
 85
(row_vector x, row_vector y):
 row_vector, 85
(vector x, matrix y): matrix, 84
(vector x, vector y): matrix, 85

```

arg
    (complex z): real, 48
asin
    (T x): R, 28
    (complex z): complex, 50
asinh
    (T x): R, 29
    (complex z): complex, 52
atan
    (T x): R, 28
    (complex z): complex, 51
atan2
    (T y, T x): R, 28
atanh
    (T x): R, 29
    (complex z): complex, 52
bernoulli
    sampling statement, 185
bernoulli_cdf
    (ints y | reals theta): real, 185
bernoulli_lccdf
    (ints y | reals theta): real, 185
bernoulli_lcdf
    (ints y | reals theta): real, 185
bernoulli_logit
    sampling statement, 186
bernoulli_logit_glm
    sampling statement, 187
bernoulli_logit_glm_lpmf
    (array[] int y | matrix x, real
        alpha, vector beta): real, 188
    (array[] int y | matrix x, vector
        alpha, vector beta): real, 189
    (array[] int y | row_vector x, real
        alpha, vector beta): real, 188
    (array[] int y | row_vector x, vector
        alpha, vector beta): real,
        188
    (int y | matrix x, real alpha,
        vector beta): real, 187
    (int y | matrix x, vector alpha,
        vector beta): real, 188
bernoulli_logit_glm_lupmf
    (array[] int y | matrix x, real
        alpha, vector beta): real, 189
    (array[] int y | matrix x, vector
        alpha, vector beta): real, 189
    (array[] int y | row_vector x, real
        alpha, vector beta): real, 188
bernoulli_logit_glm_rng
    (array[] int y | row_vector x, vec-
        tor alpha, vector beta): real,
        188
    (int y | matrix x, real alpha,
        vector beta): real, 187
    (int y | matrix x, vector alpha,
        vector beta): real, 188
bernoulli_logit_glm_rng
    (matrix x, vector alpha, vector
        beta): array[] int, 189
    (row_vector x, vector alpha, vector
        beta): array[] int, 189
bernoulli_logit_lpmf
    (ints y | reals alpha): real, 186
bernoulli_logit_lupmf
    (ints y | reals alpha): real, 186
bernoulli_logit_rng
    (reals alpha): R, 187
bernoulli_lpmf
    (ints y | reals theta): real, 185
bernoulli_lupmf
    (ints y | reals theta): real, 185
bernoulli_rng
    (reals theta): R, 186
bessel_first_kind
    (T1 x, T2 y): R, 34
    (int v, real x): real, 34
bessel_second_kind
    (T1 x, T2 y): R, 35
    (int v, real x): real, 35
beta
    (T1 x, T2 y): R, 31
    (real alpha, real beta): real, 31
    sampling statement, 265
beta_binomial
    sampling statement, 195
beta_binomial_cdf
    (ints n | ints N, reals alpha, reals
        beta): real, 196
beta_binomial_lccdf
    (ints n | ints N, reals alpha, reals
        beta): real, 196
beta_binomial_lcdf
    (ints n | ints N, reals alpha, reals
        beta): real, 196
beta_binomial_lpmf
    (ints n | ints N, reals alpha, reals
        beta): real, 195
beta_binomial_lupmf

```

```

(ints n | ints N, reals alpha, reals
      beta): real, 195
beta_binomial_rng
  (ints N, reals alpha, reals beta):
    R, 196
beta_cdf
  (reals theta | reals alpha, reals
      beta): real, 265
beta_lccdf
  (reals theta | reals alpha, reals
      beta): real, 266
beta_lcdf
  (reals theta | reals alpha, reals
      beta): real, 266
beta_lpdf
  (reals theta | reals alpha, reals
      beta): real, 265
beta_lupdf
  (reals theta | reals alpha, reals
      beta): real, 265
beta_neg_binomial
  sampling statement, 217
beta_neg_binomial_cdf
  (ints n | reals r, reals alpha,
      reals beta): real, 218
beta_neg_binomial_lccdf
  (ints n | reals r, reals alpha,
      reals beta): real, 218
beta_neg_binomial_lcdf
  (ints n | reals r, reals alpha,
      reals beta): real, 218
beta_neg_binomial_lpmf
  (ints n | reals r, reals alpha,
      reals beta): real, 218
beta_neg_binomial_lupmf
  (ints n | reals r, reals alpha,
      reals beta): real, 218
beta_neg_binomial_rng
  (reals r, reals alpha, reals beta):
    R, 218
beta_proportion
  sampling statement, 266
beta_proportion_lccdf
  (reals theta | reals mu, reals
      kappa): real, 267
beta_proportion_lcdf
  (reals theta | reals mu, reals
      kappa): real, 267
beta_proportion_lpdf
  (reals theta | reals mu, reals
      kappa): real, 267
beta_proportion_lupdf
  (reals theta | reals mu, reals
      kappa): real, 267
beta_proportion_rng
  (reals mu, reals kappa): R, 267
beta_rng
  (reals alpha, reals beta): R, 266
binary_log_loss
  (T1 x, T2 y): R, 31
  (int y, real y_hat): real, 31
binomia_cdf
  (ints n | ints N, reals theta):
    real, 191
binomia_lccdf
  (ints n | ints N, reals theta):
    real, 191
binomia_lcdf
  (ints n | ints N, reals theta):
    real, 191
binomia_lpmf
  (ints n | ints N, reals theta):
    real, 190
binomia_lupmf
  (ints n | ints N, reals theta):
    real, 190
binomial
  sampling statement, 190
binomial_logit
  sampling statement, 192
binomial_logit_glm
  sampling statement, 193
binomial_logit_glm_lpmf
  (array[] int n | array[] int N,
      matrix x, real alpha, vector
      beta): real, 194
  (array[] int n | array[] int N,
      matrix x, vector alpha, vector
      beta): real, 194
  (array[] int n | array[] int N,
      row_vector x, real alpha, vec-
      tor beta): real, 193
  (array[] int n | array[] int N,
      row_vector x, vector alpha,
      vector beta): real, 194
  (int n | int N, matrix x, real
      alpha, vector beta): real, 193
  (int n | int N, matrix x, vector
      alpha, vector beta): real, 193

```

```

alpha, vector beta): real, 193
binomial_logit_glm_lupmf
  (array[] int n | array[] int N,
   matrix x, real alpha, vector
   beta): real, 194
  (array[] int n | array[] int N,
   matrix x, vector alpha, vector
   beta): real, 195
  (array[] int n | array[] int N,
   row_vector x, real alpha, vec-
   tor beta): real, 194
  (array[] int n | array[] int N,
   row_vector x, vector alpha,
   vector beta): real, 194
  (int n | int N, matrix x, real
   alpha, vector beta): real, 193
  (int n | int N, matrix x, vector
   alpha, vector beta): real, 193
binomial_logit_lpmf
  (ints n | ints N, reals alpha):
   real, 192
binomial_logit_lupmf
  (ints n | ints N, reals alpha):
   real, 192
binomial_rng
  (ints N, reals theta): R, 191
block
  (complex_matrix x, int i, int j,
   int n_rows, int n_cols): com-
   plex_matrix, 120
  (matrix x, int i, int j, int n_rows,
   int n_cols): matrix, 82
categorical
  sampling statement, 197
categorical_logit
  sampling statement, 198
categorical_logit_glm
  sampling statement, 199
categorical_logit_glm_lpmf
  (array[] int y | matrix x, vector
   alpha, matrix beta): real, 200
  (array[] int y | row_vector x, vec-
   tor alpha, matrix beta): real,
   200
  (int y | matrix x, vector alpha,
   matrix beta): real, 200
  (int y | row_vector x, vector alpha,
   matrix beta): real, 199
categorical_logit_glm_lupmf
  (array[] int y | matrix x, vector
   alpha, matrix beta): real, 200
  (array[] int y | row_vector x, vec-
   tor alpha, matrix beta): real,
   200
  (int y | matrix x, vector alpha,
   matrix beta): real, 200
  (int y | row_vector x, vector alpha,
   matrix beta): real, 199
categorical_logit_lpmf
  (ints y | vector beta): real, 198
categorical_logit_lupmf
  (ints y | vector beta): real, 198
categorical_logit_rng
  (vector beta): int, 198
categorical_lpmf
  (ints y | vector theta): real, 198
categorical_lupmf
  (ints y | vector theta): real, 198
categorical_rng
  (vector theta): int, 198
cauchy
  sampling statement, 236
cauchy_cdf
  (reals y | reals mu, reals sigma):
   real, 236
cauchy_lccdf
  (reals y | reals mu, reals sigma):
   real, 236
cauchy_lcdf
  (reals y | reals mu, reals sigma):
   real, 236
cauchy_lpdf
  (reals y | reals mu, reals sigma):
   real, 236
cauchy_lupdf
  (reals y | reals mu, reals sigma):
   real, 236
cauchy_rng
  (reals mu, reals sigma): R, 236
cbrt
  (T x): R, 26
ceil
  (T x): R, 26
chi_square
  sampling statement, 244
chi_square_cdf
  (reals y | reals nu): real, 244
chi_square_lccdf

```

```

(real y | reals nu): real, 245
chi_square_lcdf
  (reals y | reals nu): real, 244
chi_square_lpdf
  (reals y | reals nu): real, 244
chi_square_lupdf
  (reals y | reals nu): real, 244
chi_square_rng
  (reals nu): R, 245
chol2inv
  (matrix L): matrix, 98
cholesky_decompose
  (matrix A): matrix, 102
cholesky_factor_corr_constrain
  (vectors y, int K): matrices, 164
cholesky_factor_corr_jacobian
  (vectors y, int K): matrices, 164
cholesky_factor_corr_unconstrain
  (matrices x): vectors, 164
cholesky_factor_cov_constrain
  (vectors y, int M, int N): matrices,
  164
cholesky_factor_cov_jacobian
  (vectors y, int M, int N): matrices,
  165
cholesky_factor_cov_unconstrain
  (matrices x): vectors, 165
choose
  (T1 x, T2 y): R, 34
  (int x, int y): int, 34
col
  (complex_matrix x, int n): com-
  plex_vector, 119
  (matrix x, int n): vector, 82
cols
  (complex_matrix x): int, 107
  (complex_row_vector x): int, 106
  (complex_vector x): int, 106
  (matrix x): int, 63
  (row_vector x): int, 63
  (vector x): int, 63
columns_dot_product
  (complex_matrix x, complex_matrix
  y): complex_row_vector, 114
  (complex_row_vector x, com-
  plex_row_vector y): com-
  plex_row_vector, 114
  (complex_vector x, complex_vector
  y): complex_row_vector, 114
  (matrix x, matrix y): row_vector, 71
  (row_vector x, row_vector y):
  row_vector, 71
  (vector x, vector y): row_vector, 71
columns_dot_self
  (complex_matrix x): com-
  plex_row_vector, 115
  (complex_row_vector x): com-
  plex_row_vector, 115
  (complex_vector x): com-
  plex_row_vector, 115
  (matrix x): row_vector, 72
  (row_vector x): row_vector, 72
  (vector x): row_vector, 72
complex_schur_decompose
  (complex_matrix A): tu-
  ple(complex_matrix, com-
  plex_matrix), 128
  (matrix A): tuple(complex_matrix,
  complex_matrix), 127
complex_schur_decompose_t
  (complex_matrix A): complex_matrix,
  127
  (matrix A): complex_matrix, 127
complex_schur_decompose_u
  (complex_matrix A): complex_matrix,
  127
  (matrix A): complex_matrix, 127
conj
  (Z z): Z, 48
  (complex z): complex, 48
corr_matrix_constrain
  (vectors y, int K): matrices, 165
corr_matrix_jacobian
  (vectors y, int K): matrices, 165
corr_matrix_unconstrain
  (matrices x): vectors, 165
cos
  (T x): R, 28
  (complex z): complex, 50
cosh
  (T x): R, 29
  (complex z): complex, 51
cov_exp_quad
  (array[] real x, real alpha, real
  rho): matrix, 176
  (array[] real x1, array[] real x2,
  real alpha, real rho): matrix,
  176

```

```

(row_vectors x, real alpha, real
 rho): matrix, 175
(row_vectors x1, row_vectors x2,
 real alpha, real rho): matrix,
 176
(vectors x, real alpha, real rho):
 matrix, 176
(vectors x1, vectors x2, real alpha,
 real rho): matrix, 176
cov_matrix_constrain
  (vectors y, int K): matrices, 166
cov_matrix_jacobian
  (vectors y, int K): matrices, 166
cov_matrix_unconstrain
  (matrices x): vectors, 166
crossprod
  (matrix x): matrix, 72
csr_extract
  (matrix a): tuple(vector, array[]
    int, array[] int), 130
csr_extract_u
  (matrix a): array[] int, 130
csr_extract_v
  (matrix a): array[] int, 130
csr_extract_w
  (matrix a): vector, 130
csr_matrix_times_vector
  (int m, int n, vector w, array[] int
    v, array[] int u, vector b):
    vector, 131
csr_to_dense_matrix
  (int m, int n, vector w, array[] int
    v, array[] int u): matrix, 131
cumulative_sum
  (array[] complex x): array[] real,
  124
  (array[] int x): array[] int, 87
  (array[] real x): array[] real, 87
  (complex_row_vector rv): com-
    plex_row_vector, 124
  (complex_vector v): complex_vector,
  124
  (row_vector rv): row_vector, 87
  (vector v): vector, 87
dae
  (function residual, vector ini-
    tial_state, vector ini-
    tial_state_derivative, data
    real initial_time, data array[]

    real times, ...): array[]
    vector, 150
dae_tol
  (function residual, vector ini-
    tial_state, vector ini-
    tial_state_derivative, data
    real initial_time, data array[]
    real times, data real
    rel_tol, data real abs_tol, int
    max_num_steps, ...): array[]
    vector, 150
determinant
  (matrix A): real, 97
diag_matrix
  (complex_vector x): complex_matrix,
  119
  (vector x): matrix, 79
diag_post_multiply
  (complex_matrix m, com-
    plex_row_vector v): com-
    plex_matrix, 116
  (complex_matrix m, complex_vector
    v): complex_matrix, 116
  (matrix m, row_vector rv): matrix,
  74
  (matrix m, vector v): matrix, 74
diag_pre_multiply
  (complex_row_vector v, com-
    plex_matrix m): complex_matrix,
  116
  (complex_vector v, complex_matrix
    m): complex_matrix, 116
  (row_vector rv, matrix m): matrix,
  74
  (vector v, matrix m): matrix, 74
diagonal
  (complex_matrix x): complex_vector,
  119
  (matrix x): vector, 79
digamma
  (T x): R, 32
dims
  (T x): array[] int, 58
dirichlet
  sampling statement, 288
dirichlet_lpdf
  (vectors theta | vectors alpha):
    real, 288
dirichlet_lupdf

```

```

(vectors theta | vectors alpha):
    real, 289
dirichlet_multinomial
    sampling statement, 221
dirichlet_multinomial_lpmf
    (array[] int y | vector alpha):
        real, 221
dirichlet_multinomial_lupmf
    (array[] int y | vector alpha):
        real, 221
dirichlet_multinomial_rng
    (vector alpha, int N): array[] int,
        222
dirichlet_rng
    (vector alpha): vector, 289
discrete_range
    sampling statement, 201
discrete_range_cdf
    (ints n | ints N, reals theta):
        real, 201
discrete_range_lccdf
    (ints n | ints N, reals theta):
        real, 202
discrete_range_lcddf
    (ints n | ints N, reals theta):
        real, 201
discrete_range_lcdf
    (ints n | ints N, reals theta):
        real, 201
discrete_range_lpmf
    (ints y | ints l, ints u): real, 201
discrete_range_lupmf
    (ints y | ints l, ints u): real, 201
discrete_range_rng
    (ints l, ints u): ints, 202
distance
    (row_vector x, row_vector y): real,
        56
    (row_vector x, vector y): real, 56
    (vector x, row_vector y): real, 56
    (vector x, vector y): real, 56
dot_product
    (complex_row_vector x, com-
        plex_row_vector y): complex,
        114
    (complex_row_vector x, com-
        plex_vector y): complex, 114
    (complex_vector x, com-
        plex_row_vector y): complex,
        114
    (complex_vector x, complex_vector
        y): complex, 114
    (row_vector x, row_vector y): real,
        71
    (row_vector x, vector y): real, 71
    (vector x, row_vector y): real, 71
    (vector x, vector y): real, 71
dot_self
    (complex_row_vector x): complex, 115
    (complex_vector x): complex, 115
    (row_vector x): real, 72
    (vector x): real, 72
double_exponential
    sampling statement, 237
double_exponential_cdf
    (reals y | reals mu, reals sigma):
        real, 238
double_exponential_lccdf
    (reals y | reals mu, reals sigma):
        real, 238
double_exponential_lcddf
    (reals y | reals mu, reals sigma):
        real, 238
double_exponential_lcdf
    (reals y | reals mu, reals sigma):
        real, 238
double_exponential_lpdf
    (reals y | reals mu, reals sigma):
        real, 238
double_exponential_lupdf
    (reals y | reals mu, reals sigma):
        real, 238
double_exponential_rng
    (reals mu, reals sigma): R, 238
e
    (): real, 16
eigendecompose
    (complex_matrix A): tu-
        ple(complex_matrix, com-
            plex_vector), 125
    (matrix A): tuple(complex_matrix,
        complex_vector), 99
eigendecompose_sym
    (complex_matrix A): tu-
        ple(complex_matrix, com-
            plex_vector), 125
    (matrix A): tuple(matrix, vector),
        100
eigenvalues
    (complex_matrix A): complex_vector,
        125
    (matrix A): complex_vector, 99
eigenvalues_sym
    (complex_matrix A): complex_vector,
        125

```

```

125
  (matrix A): vector, 99
eigenvectors
  (complex_matrix A): complex_matrix,
125
  (matrix A): complex_matrix, 99
eigenvectors_sym
  (complex_matrix A): complex_matrix,
125
  (matrix A): matrix, 100
erf
  (T x): R, 30
erfc
  (T x): R, 30
exp
  (T x): R, 26
  (complex z): complex, 49
exp2
  (T x): R, 26
exp_mod_normal
  sampling statement, 232
exp_mod_normal_cdf
  (reals y | reals mu, reals sigma,
  reals lambda): real, 232
exp_mod_normal_lccdf
  (reals y | reals mu, reals sigma,
  reals lambda): real, 232
exp_mod_normal_lcddf
  (reals y | reals mu, reals sigma,
  reals lambda): real, 232
exp_mod_normal_lpdf
  (reals y | reals mu, reals sigma,
  reals lambda): real, 232
exp_mod_normal_lupdf
  (reals y | reals mu, reals sigma,
  reals lambda): real, 232
exp_mod_normal_rng
  (reals mu, reals sigma, reals
  lambda): R, 233
expm1
  (T x): R, 38
exponential
  sampling statement, 247
exponential_cdf
  (reals y | reals beta): real, 248
exponential_lccdf
  (reals y | reals beta): real, 248
exponential_lcddf
  (reals y | reals beta): real, 248
exponential_lpdf
  (reals y | reals beta): real, 248
exponential_lupdf
  (reals y | reals beta): real, 248
exponential_rng
  (reals beta): R, 248
falling_factorial
  (T1 x, T2 y): R, 36
  (real x, real n): real, 36
fatal_error
  (T1 x1,..., TN xN): void, 5
fdim
  (T1 x, T2 y): R, 24
  (real x, real y): real, 24
fft
  (complex_vector v): complex_vector,
123
fft2
  (complex_matrix m): complex_matrix,
123
floor
  (T x): R, 25
fma
  (real x, real y, real z): real, 38
fmax
  (T1 x, T2 y): R, 25
  (real x, real y): real, 25
fmin
  (T1 x, T2 y): R, 24
  (real x, real y): real, 24
fmod
  (T1 x, T2 y): R, 25
  (real x, real y): real, 25
frechet
  sampling statement, 252
frechet_cdf
  (reals y | reals alpha, reals
  sigma): real, 252
frechet_lccdf
  (reals y | reals alpha, reals
  sigma): real, 253
frechet_lcddf
  (reals y | reals alpha, reals
  sigma): real, 253
frechet_lpdf
  (reals y | reals alpha, reals
  sigma): real, 252
frechet_lupdf

```

```

        (reals y | reals alpha, reals
         sigma): real, 252
frechet_rng
        (reals alpha, reals sigma): R, 253
gamma
        sampling statement, 249
gamma_cdf
        (reals y | reals alpha, reals beta):
         real, 249
gamma_lccdf
        (reals y | reals alpha, reals beta):
         real, 249
gamma_lcdf
        (reals y | reals alpha, reals beta):
         real, 249
gamma_lpdf
        (reals y | reals alpha, reals beta):
         real, 249
gamma_lupdf
        (reals y | reals alpha, reals beta):
         real, 249
gamma_p
        (T1 x, T2 y): R, 33
        (real a, real z): real, 33
gamma_q
        (T1 x, T2 y): R, 34
        (real a, real z): real, 33
gamma_rng
        (reals alpha, reals beta): R, 249
gaussian_dlm_obs
        sampling statement, 285
gaussian_dlm_obs_lpdf
        (matrix y | matrix F, matrix G,
         matrix V, matrix W, vector m0,
         matrix C0): real, 285
        (matrix y | matrix F, matrix G,
         vector V, matrix W, vector m0,
         matrix C0): real, 286
gaussian_dlm_obs_lupdf
        (matrix y | matrix F, matrix G,
         matrix V, matrix W, vector m0,
         matrix C0): real, 285
        (matrix y | matrix F, matrix G,
         vector V, matrix W, vector m0,
         matrix C0): real, 286
generalized_inverse
        (matrix A): matrix, 99
get_imag
        (T x): T_demoted, 118
        (complex z): real, 44
get_real
        (T x): T_demoted, 118
        (complex z): real, 44
gp_dot_prod_cov
        (array[] real x, real sigma): ma-
         trix, 89
        (array[] real x1, array[] real x2,
         real sigma): matrix, 89
        (vectors x, real sigma): matrix, 89
        (vectors x1, vectors x2, real
         sigma): matrix, 89
gp_exp_quad_cov
        (array[] real x, real sigma, real
         length_scale): matrix, 88
        (array[] real x1, array[] real x2,
         real sigma, real length_scale):
         matrix, 88
        (vectors x, real sigma, array[] real
         length_scale): matrix, 88
        (vectors x, real sigma, real
         length_scale): matrix, 88
        (vectors x1, vectors x2, real sigma,
         array[] real length_scale):
         matrix, 89
        (vectors x1, vectors x2, real sigma,
         real length_scale): matrix, 88
gp_exponential_cov
        (array[] real x, real sigma, real
         length_scale): matrix, 90
        (array[] real x1, array[] real x2,
         real sigma, real length_scale):
         matrix, 90
        (vectors x, real sigma, array[] real
         length_scale): matrix, 90
        (vectors x, real sigma, real
         length_scale): matrix, 90
        (vectors x1, vectors x2, real sigma,
         array[] real length_scale):
         matrix, 91
        (vectors x1, vectors x2, real sigma,
         real length_scale): matrix, 90
gp_matern32_cov
        (array[] real x, real sigma, real
         length_scale): matrix, 91
        (array[] real x1, array[] real x2,
         real sigma, real length_scale):
         matrix, 91
        (vectors x, real sigma, array[] real
         length_scale): matrix, 91

```

```

length_scale): matrix, 92
(vectors x, real sigma, real
length_scale): matrix, 91
(vectors x1, vectors x2, real sigma,
array[] real length_scale):
matrix, 92
(vectors x1, vectors x2, real sigma,
real length_scale): matrix, 92
gp_matern52_cov
(array[] real x, real sigma, real
length_scale): matrix, 92
(array[] real x1, array[] real x2,
real sigma, real length_scale):
matrix, 92
(vectors x, real sigma, array[] real
length_scale): matrix, 93
(vectors x, real sigma, real
length_scale): matrix, 93
(vectors x1, vectors x2, real sigma,
array[] real length_scale):
matrix, 93
(vectors x1, vectors x2, real sigma,
real length_scale): matrix, 93
gp_periodic_cov
(array[] real x, real sigma, real
length_scale, real period):
matrix, 94
(array[] real x1, array[] real x2,
real sigma, real length_scale,
real period): matrix, 94
(vectors x, real sigma, real
length_scale, real period):
matrix, 94
(vectors x1, vectors x2, real sigma,
real length_scale, real pe-
riod): matrix, 94
gumbel
sampling statement, 240
gumbel_cdf
(real y | reals mu, reals beta):
real, 240
gumbel_lccdf
(real y | reals mu, reals beta):
real, 240
gumbel_lcdf
(real y | reals mu, reals beta):
real, 240
gumbel_lpdf
(real y | reals mu, reals beta):
real, 240
real, 240
gumbel_lupdf
(real y | reals mu, reals beta):
real, 240
gumbel_rng
(reals mu, reals beta): R, 240
head
(array[] T sv, int n): array[] T, 83
(complex_row_vector rv, int n):
complex_row_vector, 120
(complex_vector v, int n): com-
plex_vector, 120
(row_vector rv, int n): row_vector,
83
(vector v, int n): vector, 83
hmm_hidden_state_prob
(matrix log_omega, matrix Gamma,
vector rho): matrix, 301
hmm_latent_rng
(matrix log_omega, matrix Gamma,
vector rho): array[] int, 301
hmm_marginal
(matrix log_omega, matrix Gamma,
vector rho): real, 300
hypergeometric
sampling statement, 197
hypergeometric_1F0
(real a, real z): real, 41
hypergeometric_2F1
(real a1, real a2, real b1, real z):
real, 41
hypergeometric_3F2
(T1 a, T2 b, real z): real, 42
hypergeometric_lpmf
(int n | int N, int a, int b):
real, 197
hypergeometric_lupmf
(int n | int N, int a, int b):
real, 197
hypergeometric_pFq
(T1 a, T2 b, real z): real, 42
hypergeometric_rng
(int N, int a, int2 b): int, 197
hypot
(T1 x, T2 y): R, 28
(real x, real y): real, 28
identity_matrix_matrix
(int k): matrix, 80
inc_beta

```

```

(real alpha, real beta, real x):
    real, 32
int_step
    (int x): int, 8
    (real x): int, 8
integrate_1d
    (function integrand, real a, real
        b, array[] real theta, array[]
        real x_r, array[] int x_i):
            real, 153
    (function integrand, real a, real
        b, array[] real theta, array[]
        real x_r, array[] int x_i, real
        relative_tolerance): real, 153
integrate_ode
    (function ode, array[] real ini-
        tial_state, real initial_time,
        array[] real times, array[]
        real theta, array[] real x_r,
        array[] int x_i): array[,]
            real, 170
integrate_ode_adams
    (function ode, array[] real ini-
        tial_state, real initial_time,
        array[] real times, array[]
        real theta, data array[] real
        x_r, data array[] int x_i):
            array[,] real, 170
    (function ode, array[] real ini-
        tial_state, real initial_time,
        array[] real times, array[]
        real theta, data array[] int
        x_i, data real rel_tol,
        data real abs_tol, data int
        max_num_steps): array[,] real,
            171
integrate_ode_bdf
    (function ode, array[] real ini-
        tial_state, real initial_time,
        array[] real times, array[]
        real theta, data array[] real
        x_r, data array[] int x_i):
            array[,] real, 171
    (function ode, array[] real ini-
        tial_state, real initial_time,
        array[] real times, array[]
        real theta, data array[] int
        x_i, data array[] int
        x_i, data real rel_tol,
        data real abs_tol, data int
        max_num_steps): array[,] real,
            171
x_i, data real rel_tol,
data real abs_tol, data int
max_num_steps): array[,] real,
171
integrate_ode_rk45
    (function ode, array[] real ini-
        tial_state, real initial_time,
        array[] real times, array[]
        real theta, array[] real x_r,
        array[] int x_i): array[,]
            real, 170
    (function ode, array[] real ini-
        tial_state, real initial_time,
        array[] real times, array[]
        real theta, array[] real
        x_r, array[] int x_i, real
        rel_tol, real abs_tol, int
        max_num_steps): array[,] real,
            170
inv
    (T x): R, 27
inv_chi_square
    sampling statement, 245
inv_chi_square_cdf
    (reals y | reals nu): real, 245
inv_chi_square_lccdf
    (reals y | reals nu): real, 246
inv_chi_square_lcddf
    (reals y | reals nu): real, 246
inv_chi_square_lpdf
    (reals y | reals nu): real, 245
inv_chi_square_lupdf
    (reals y | reals nu): real, 245
inv_chi_square_rng
    (reals nu): R, 246
inv_cloglog
    (T x): R, 30
inv_erfc
    (T x): R, 30
inv_fft
    (complex_vector u): complex_vector,
        123
inv_fft2
    (complex_matrix m): complex_matrix,
        124
inv_gamma
    sampling statement, 250
inv_gamma_cdf
    (reals y | reals alpha, reals beta):

```

```

real, 250
inv_gamma_lccdf
  (reals y | reals alpha, reals beta):
    real, 250
inv_gamma_lcdf
  (reals y | reals alpha, reals beta):
    real, 250
inv_gamma_lpdf
  (reals y | reals alpha, reals beta):
    real, 250
inv_gamma_lupdf
  (reals y | reals alpha, reals beta):
    real, 250
inv_gamma_rng
  (reals alpha, reals beta): R, 250
inv_inc_beta
  (real alpha, real beta, real p):
    real, 32
inv_logit
  (T x): R, 30
inv_phi
  (T x): R, 30
inv_sqrt
  (T x): R, 27
inv_square
  (T x): R, 27
inv_wishart
  sampling statement, 295
inv_wishart_cholesky_lpdf
  (matrix L_W | real nu, matrix L_S):
    real, 297
inv_wishart_lpdf
  (matrix W | real nu, matrix Sigma):
    real, 295
inv_wishart_lupdf
  (matrix L_W | real nu, matrix L_S):
    real, 297
  (matrix W | real nu, matrix Sigma):
    real, 295
inv_wishart_rng
  (real nu, matrix L_S): matrix, 297
  (real nu, matrix Sigma): matrix, 296
inverse
  (matrix A): matrix, 98
inverse_spd
  (matrix A): matrix, 98
is_inf
  (real x): int, 22
is_nan
  (real x): int, 22
lambert_w0
  (T x): R, 41
lambert_wm1
  (T x): R, 41
lbeta
  (T1 x, T2 y): R, 32
  (real alpha, real beta): real, 32
lchoose
  (T1 x, T2 y): R, 37
  (real x, real y): real, 36
lexp
  (T1 x, T2 y): R, 38
  (real x, int y): real, 38
lgamma
  (T x): R, 32
linspace_array
  (int n, data real lower, data real
    upper): array[] real, 80
linspace_int_array
  (int n, int lower, int upper):
    array[] real, 80
linspace_row_vector
  (int n, data real lower, data real
    upper): row_vector, 80
linspace_vector
  (int n, data real lower, data real
    upper): vector, 80
lkj_corr
  sampling statement, 291
lkj_corr_cholesky
  sampling statement, 292
lkj_corr_cholesky_lpdf
  (matrix L | real eta): real, 292
lkj_corr_cholesky_lupdf
  (matrix L | real eta): real, 292
lkj_corr_cholesky_rng
  (int K, real eta): matrix, 292
lkj_corr_lpdf
  (matrix y | real eta): real, 291
lkj_corr_lupdf
  (matrix y | real eta): real, 291
lkj_corr_rng
  (int K, real eta): matrix, 291
lmgamma
  (T1 x, T2 y): R, 33
  (int n, real x): real, 33
lmultiply
  (T1 x, T2 y): R, 38

```

```

(real x, real y): real, 38
log
  (T x): R, 26
  (complex z): complex, 49
log10
  (): real, 16
  (T x): R, 27
  (complex z): complex, 49
log1m
  (T x): R, 39
log1m_exp
  (T x): R, 39
log1m_inv_logit
  (T x): R, 40
log1p
  (T x): R, 38
log1p_exp
  (T x): R, 39
log2
  (): real, 16
  (T x): R, 27
log_determinant
  (matrix A): real, 98
log_diff_exp
  (T1 x, T2 y): R, 39
  (real x, real y): real, 39
log_falling_factorial
  (real x, real n): real, 37
log_inv_logit
  (T x): R, 40
log_inv_logit_diff
  (T1 x, T2 y): R, 40
log_mix
  (T1 thetas, T2 lps): real, 40
  (real theta, real lp1, real lp2):
    real, 39
log_modified_bessel_first_kind
  (T1 x, T2 y): R, 35
  (real v, real z): real, 35
log_rising_factorial
  (T1 x, T2 y): R, 37
  (real x, real n): real, 37
log_softmax
  (vector x): vector, 86
log_sum_exp
  (T1 x, T2 y): R, 40
  (array[] real x): real, 54
  (matrix x): real, 75
  (row_vector x): real, 75
  (vector x): real, 75
logistic
  sampling statement, 239
logistic_cdf
  (reals y | reals mu, reals sigma):
    real, 239
logistic_lccdf
  (reals y | reals mu, reals sigma):
    real, 239
logistic_lcdf
  (reals y | reals mu, reals sigma):
    real, 239
logistic_lpdf
  (reals y | reals mu, reals sigma):
    real, 239
logistic_lupdf
  (reals y | reals mu, reals sigma):
    real, 239
logistic_rng
  (reals mu, reals sigma): R, 239
logit
  (T x): R, 29
loglogistic
  sampling statement, 254
loglogistic_cdf
  (reals y | reals alpha, reals beta):
    real, 255
loglogistic_lpdf
  (reals y | reals alpha, reals beta):
    real, 255
loglogistic_rng
  (reals alpha, reals beta): R, 255
lognormal
  sampling statement, 243
lognormal_cdf
  (reals y | reals mu, reals sigma):
    real, 243
lognormal_lccdf
  (reals y | reals mu, reals sigma):
    real, 244
lognormal_lcdf
  (reals y | reals mu, reals sigma):
    real, 243
lognormal_lpdf
  (reals y | reals mu, reals sigma):
    real, 243
lognormal_lupdf
  (reals y | reals mu, reals sigma):
    real, 243

```

```

lognormal_rng
    (reals mu, reals sigma):  R, 244
lower_bound_constraint
    (reals y, reals lb):  reals, 158
lower_bound_jacobian
    (reals y, reals lb):  reals, 158
lower_bound_unconstrain
    (reals x, reals lb):  reals, 159
lower_upper_bound_constraint
    (reals y, reals lb, reals ub):
        reals, 159
lower_upper_bound_jacobian
    (reals y, reals lb, reals ub):
        reals, 160
lower_upper_bound_unconstrain
    (reals x, reals lb, reals ub):
        reals, 160
machine_precision
    ():  real, 17
map_rect
    (F f, vector phi, array[] vector
        theta, data array[,] real x_r,
        data array[,] int x_i):  vector,
        157
matrix_exp
    (matrix A): matrix, 97
matrix_exp_multiply
    (matrix A, matrix B): matrix, 97
matrix_power
    (matrix A, int B): matrix, 97
max
    (array[] int x):  int, 53
    (array[] real x):  real, 53
    (int x, int y):  int, 9
    (matrix x):  real, 75
    (row_vector x):  real, 75
    (vector x):  real, 75
mdivide_left_spd
    (matrix A, matrix B): vector, 96
    (matrix A, vector b):  matrix, 96
mdivide_left_tri_low
    (matrix A, matrix B): matrix, 95
    (matrix A, vector b):  vector, 95
mdivide_right_spd
    (matrix B, matrix A): matrix, 96
    (row_vector b, matrix A):
        row_vector, 96
mdivide_right_tri_low
    (matrix B, matrix A): matrix, 96
    (row_vector b, matrix A):
        row_vector, 96
mean
    (array[] real x):  real, 54
    (matrix x):  real, 76
    (row_vector x):  real, 76
    (vector x):  real, 76
min
    (array[] int x):  int, 53
    (array[] real x):  real, 53
    (int x, int y):  int, 9
    (matrix x):  real, 75
    (row_vector x):  real, 75
    (vector x):  real, 75
modified_bessel_first_kind
    (T1 x, T2 y):  R, 35
    (int v, real z):  real, 35
modified_bessel_second_kind
    (T1 x, T2 y):  R, 36
    (int v, real z):  real, 36
multi_gp
    sampling statement, 280
multi_gp_cholesky
    sampling statement, 280
multi_gp_cholesky_lpdf
    (matrix y | matrix L, vector w):
        real, 280
multi_gp_cholesky_lupdf
    (matrix y | matrix L, vector w):
        real, 281
multi_gp_lpdf
    (matrix y | matrix Sigma, vector w):
        real, 280
multi_gp_lupdf
    (matrix y | matrix Sigma, vector w):
        real, 280
multi_normal
    sampling statement, 273
multi_normal_cholesky
    sampling statement, 277
multi_normal_cholesky_lpdf
    (row_vectors y | row_vectors mu,
        matrix L): real, 278
    (row_vectors y | vectors mu, matrix
        L): real, 278
    (vectors y | row_vectors mu, matrix
        L): real, 277
    (vectors y | vectors mu, matrix L):
        real, 277

```

```

multi_normal_cholesky_lupdf
  (row_vectors y | row_vectors mu,
   matrix L): real, 278
  (row_vectors y | vectors mu, matrix
   L): real, 278
  (vectors y | row_vectors mu, matrix
   L): real, 278
  (vectors y | vectors mu, matrix L):
   real, 277
multi_normal_cholesky_rng
  (row_vector mu, matrix L): vector,
   279
  (row_vectors mu, matrix L): vectors,
   279
  (vector mu, matrix L): vector, 278
  (vectors mu, matrix L): vectors, 279
multi_normal_lpdf
  (row_vectors y | row_vectors mu,
   matrix Sigma): real, 274
  (row_vectors y | vectors mu, matrix
   Sigma): real, 274
  (vectors y | row_vectors mu, matrix
   Sigma): real, 274
  (vectors y | vectors mu, matrix
   Sigma): real, 273
multi_normal_lupdf
  (row_vectors y | row_vectors mu,
   matrix Sigma): real, 274
  (row_vectors y | vectors mu, matrix
   Sigma): real, 274
  (vectors y | row_vectors mu, matrix
   Sigma): real, 274
  (vectors y | vectors mu, matrix
   Sigma): real, 273
multi_normal_prec
  sampling statement, 275
multi_normal_prec_lpdf
  (row_vectors y | row_vectors mu,
   matrix Omega): real, 276
  (row_vectors y | vectors mu, matrix
   Omega): real, 276
  (vectors y | row_vectors mu, matrix
   Omega): real, 276
  (vectors y | vectors mu, matrix
   Omega): real, 275
multi_normal_prec_lupdf
  (row_vectors y | row_vectors mu,
   matrix Omega): real, 276
  (row_vectors y | vectors mu, matrix
   Sigma): real, 276
  (vectors y | row_vectors mu, matrix
   Omega): real, 276
  (vectors y | vectors mu, matrix
   Sigma): real, 275
  (vectors y | vectors mu, matrix
   Omega): real, 276
  (vectors y | vectors mu, matrix
   Sigma): real, 275
multi_normal_rng
  (row_vector mu, matrix Sigma):
   vector, 275
  (row_vectors mu, matrix Sigma):
   vectors, 275
  (vector mu, matrix Sigma): vector,
   274
  (vectors mu, matrix Sigma): vectors,
   275
multi_student_t
  sampling statement, 281
multi_student_t_cholesky
  sampling statement, 283
multi_student_t_cholesky_lpdf
  (vectors y | real nu, vectors mu,
   matrix L): real, 284
multi_student_t_cholesky_lupdf
  (vectors y | real nu, vectors mu,
   matrix L): real, 284
multi_student_t_cholesky_rng
  (real nu, row_vectors mu, matrix L):
   vector, 284
  (real nu, vector mu, matrix L):
   vector, 284
multi_student_t_lpdf
  (row_vectors y | real nu,
   row_vectors mu, matrix Sigma):
   real, 282
  (row_vectors y | real nu, vectors
   mu, matrix Sigma): real, 282
  (vectors y | real nu, row_vectors
   mu, matrix Sigma): real, 281
  (vectors y | real nu, vectors mu,
   matrix Sigma): real, 281
multi_student_t_lupdf
  (row_vectors y | real nu,
   row_vectors mu, matrix Sigma):
   real, 282
  (row_vectors y | real nu, vectors
   mu, matrix Sigma): real, 282
  (vectors y | real nu, row_vectors
   mu, matrix Sigma): real, 282
  (vectors y | real nu, vectors mu,
   matrix Sigma): real, 281

```

```

multi_student_t_rng
  (real nu, row_vector mu, matrix
   Sigma): vector, 283
  (real nu, row_vectors mu, matrix
   Sigma): vectors, 283
  (real nu, vector mu, matrix Sigma):
   vector, 282
  (real nu, vectors mu, matrix Sigma):
   vectors, 283
multinomial
  sampling statement, 219
multinomial_logit
  sampling statement, 220
multinomial_logit_lpmf
  (array[] int y | vector gamma):
   real, 220
multinomial_logit_lupmf
  (array[] int y | vector gamma):
   real, 220
multinomial_logit_rng
  (vector gamma, int N): array[] int,
   221
multinomial_lpmf
  (array[] int y | vector theta):
   real, 219
multinomial_lupmf
  (array[] int y | vector theta):
   real, 219
multinomial_rng
  (vector theta, int N): array[] int,
   219
multiply_lower_tri_self_transpose
  (matrix x): matrix, 74
neg_binomial
  sampling statement, 207
neg_binomial_2
  sampling statement, 209
neg_binomial_2_cdf
  (ints n | reals mu, reals phi):
   real, 209
neg_binomial_2_lccdf
  (ints n | reals mu, reals phi):
   real, 209
neg_binomial_2_lcdf
  (ints n | reals mu, reals phi):
   real, 209
neg_binomial_2_log
  sampling statement, 210
neg_binomial_2_log_glm
  sampling statement, 211
neg_binomial_2_log_glm_lpmf
  (array[] int y | matrix x, real
   alpha, vector beta, real phi):
   real, 212
  (array[] int y | matrix x, vector
   alpha, vector beta, real phi):
   real, 213
  (array[] int y | row_vector x, real
   alpha, vector beta, real phi):
   real, 212
  (array[] int y | row_vector x, vector
   alpha, vector beta, real phi):
   real, 212
  (int y | matrix x, real alpha, vec-
   tor beta, real phi): real,
   211
  (int y | matrix x, vector alpha,
   vector beta, real phi): real,
   211
neg_binomial_2_log_glm_lupmf
  (array[] int y | matrix x, real
   alpha, vector beta, real phi):
   real, 213
  (array[] int y | matrix x, vector
   alpha, vector beta, real phi):
   real, 213
  (array[] int y | row_vector x, real
   alpha, vector beta, real phi):
   real, 212
  (array[] int y | row_vector x, vector
   alpha, vector beta, real phi):
   real, 212
  (int y | matrix x, real alpha, vec-
   tor beta, real phi): real,
   211
  (int y | matrix x, vector alpha,
   vector beta, real phi): real,
   212
neg_binomial_2_log_lpmf
  (ints n | reals eta, reals phi):
   real, 210
neg_binomial_2_log_lupmf
  (ints n | reals eta, reals phi):
   real, 210
neg_binomial_2_log_rng
  (reals eta, reals phi): R, 210
neg_binomial_2_lpmf
  (ints n | reals mu, reals phi):

```

```

real, 209
neg_binomial_2_lupmf
(ints n | reals mu, reals phi):
    real, 209
neg_binomial_2_rng
    (reals mu, reals phi): R, 210
neg_binomial_cdf
    (ints n | reals alpha, reals beta):
        real, 208
neg_binomial_lccdf
    (ints n | reals alpha, reals beta):
        real, 208
neg_binomial_lcdf
    (ints n | reals alpha, reals beta):
        real, 208
neg_binomial_lpmf
    (ints n | reals alpha, reals beta):
        real, 207
neg_binomial_lupmf
    (ints n | reals alpha, reals beta):
        real, 207
neg_binomial_rng
    (reals alpha, reals beta): R, 208
negative_infinity
    (): real, 17
norm
    (complex z): real, 48
norm1
    (array[] real x): real, 55
    (row_vector x): real, 55
    (vector x): real, 55
norm2
    (array[] real x): real, 56
    (row_vector x): real, 56
    (vector x): real, 55
normal
    sampling statement, 225
normal_cdf
    (reals y | reals mu, reals sigma):
        real, 225
normal_id_glm
    sampling statement, 228
normal_id_glm_lpdf
    (real y | matrix x, real alpha,
        vector beta, real sigma): real,
        228
    (real y | matrix x, real alpha, vec-
        tor beta, vector sigma): real,
        229
    (real y | matrix x, vector alpha,
        vector beta, vector sigma): real,
        229
    (real y | matrix x, real alpha,
        vector beta, real sigma): real,
        230
    (vector y | matrix x, real alpha,
        vector beta, vector sigma): real,
        231
    (vector y | matrix x, vector alpha,
        vector beta, real sigma): real,
        230
    (vector y | matrix x, vector alpha,
        vector beta, vector sigma): real,
        231
    (vector y | matrix x, real alpha,
        vector beta, vector sigma): real,
        230
    (vector y | matrix x, vector alpha,
        vector beta, real sigma): real,
        231
    (vector y | row_vector x, real al-
        pha, vector beta, real sigma):
        real, 229
    (vector y | row_vector x, vector al-
        pha, vector beta, real sigma):
        real, 230
normal_id_glm_lupdf
    (real y | matrix x, real alpha,
        vector beta, real sigma): real,
        228
    (real y | matrix x, real alpha, vec-
        tor beta, vector sigma): real,
        229
    (real y | matrix x, vector alpha,
        vector beta, real sigma): real,
        229
    (real y | matrix x, real alpha,
        vector beta, vector sigma): real,
        229
    (real y | matrix x, real alpha,
        vector beta, real sigma): real,
        230
    (real y | matrix x, vector alpha,
        vector beta, vector sigma): real,
        229
    (vector y | matrix x, real alpha,
        vector beta, vector sigma): real,
        229
    (vector y | matrix x, real alpha,
        vector beta, real sigma): real,
        230
    (vector y | matrix x, vector alpha,
        vector beta, vector sigma): real,
        231
    (vector y | matrix x, real alpha,
        vector beta, vector sigma): real,
        231
    (vector y | matrix x, vector alpha,
        vector beta, vector sigma): real,
        231
    (vector y | matrix x, vector alpha,
        vector beta, vector sigma): real,
        231

```

```

(vector y | row_vector x, real al-
pha, vector beta, real sigma):
real, 230
(vector y | row_vector x, vector al-
pha, vector beta, real sigma):
real, 230
normal_lccdf
(reals y | reals mu, reals sigma):
real, 226
normal_lcdf
(reals y | reals mu, reals sigma):
real, 225
normal_lpdf
(reals y | reals mu, reals sigma):
real, 225
normal_lupdf
(reals y | reals mu, reals sigma):
real, 225
normal_rng
(reals mu, reals sigma): R, 226
not_a_number
(): real, 17
num_elements
(array[] T x): int, 58
(complex_matrix x): int, 106
(complex_row_vector x): int, 106
(complex_vector x): int, 106
(matrix x): int, 63
(row_vector x): int, 63
(vector x): int, 63
ode_adams
(function ode, vector initial_state,
real initial_time, array[] real
times, ...): array[] vector,
146
ode_adams_tol
(function ode, vector initial_state,
real initial_time, array[] real
times, data real rel_tol,
data real abs_tol, data int
max_num_steps, ...): array[]
vector, 146
ode_bdf
(function ode, vector initial_state,
real initial_time, array[] real
times, ...): array[] vector,
146
ode_bdf_tol
(function ode, vector initial_state,
real initial_time, array[] real
times, data real rel_tol,
data real abs_tol, data int
max_num_steps, ...): array[]
vector, 146
ode_ckrk
(function ode, array[] real ini-
tial_state, real initial_time,
array[] real times, ...): ar-
ray[] vector, 145
ode_ckrk_tol
(function ode, vector initial_state,
real initial_time, ar-
ray[] real times, data real
rel_tol, data real abs_tol, int
max_num_steps, ...): array[]
vector, 146
ode_rk45
(function ode, array[] real ini-
tial_state, real initial_time,
array[] real times, ...): ar-
ray[] vector, 145
ode_rk45_tol
(function ode, vector initial_state,
real initial_time, ar-
ray[] real times, data real
rel_tol, data real abs_tol, int
max_num_steps, ...): array[]
vector, 145
offset_multiplier_constrain
(reals y, reals offset, reals mult):
reals, 160
offset_multiplier_jacobian
(reals y, reals offset, reals mult):

```

```

    reals, 160
offset_multiplier_unconstrain
    (reals x, reals offset, reals mult):
        reals, 160
one_hot_array
    (int n, int k): array[] real, 81
one_hot_int_array
    (int n, int k): array[] int, 81
one_hot_row_vector
    (int n, int k): row_vector, 81
one_hot_vector
    (int n, int k): vector, 81
ones_array
    (int n): array[] real, 81
ones_int_array
    (int n): array[] int, 81
ones_row_vector
    (int n): row_vector, 81
ones_vector
    (int n): vector, 81
operator/
    (complex_matrix B, complex_matrix
        A): complex_matrix, 125
    (complex_row_vector b, com-
        plex_matrix A): com-
            plex_row_vector, 124
operator_add
    (complex x, complex y): complex, 45
    (complex x, complex_matrix y):
        complex_matrix, 110
    (complex x, complex_vector y):
        complex_vector, 110
    (complex x, row_complex_vector y):
        row_complex_vector, 110
    (complex_matrix x, complex y):
        complex_matrix, 110
    (complex_matrix x, complex_matrix
        y): complex_matrix, 108
    (complex_vector x, complex y):
        complex_vector, 110
    (complex_vector x, complex_vector
        y): complex_vector, 108
    (int x): int, 8
    (int x, int y): int, 7
    (matrix x, matrix y): matrix, 65
    (matrix x, real y): matrix, 67
    (real x): real, 23
    (real x, matrix y): matrix, 67
    (real x, real y): real, 22
    (real x, row_vector y): row_vector, 66
    (real x, vector y): vector, 66
    (row_complex_vector x, complex y):
        row_complex_vector, 110
    (row_complex_vector x,
        row_complex_vector y):
        row_complex_vector, 108
    (row_vector x, real y): row_vector,
        66
    (row_vector x, row_vector y):
        row_vector, 65
    (vector x, real y): vector, 66
    (vector x, vector y): vector, 65
operator_add
    (complex z): complex, 45
operator_assign
    (complex x, complex y): void, 47
operator_compound_add
    (T x, U y): void, 140
    (complex x, complex y): void, 47
operator_compound_divide
    (T x, U y): void, 141
    (complex x, complex y): void, 47
operator_compound_elt_divide
    (T x, U y): void, 141
operator_compound_elt_multiply
    (T x, U y): void, 141
operator_compound_multiply
    (complex x, complex y): void, 47
operator_compound_subtract
    (T x, U y): void, 140
    (complex x, complex y): void, 47
operator_divide
    (complex x, complex y): complex, 45
    (complex_matrix x, complex y):
        complex_matrix, 111
    (complex_vector x, complex y):
        complex_vector, 111
    (int x, int y): int, 7
    (matrix B, matrix A): matrix, 95
    (matrix x, real y): matrix, 68
    (real x, real y): real, 23
    (row_complex_vector x, complex y):
        row_complex_vector, 111
    (row_vector b, matrix A):
        row_vector, 95

```

```

(row_vector x, real y):  row_vector,
  68
(vector x, real y):  vector, 67
operator_elt_divide
  (complex x, complex_matrix y):
    complex_matrix, 113
  (complex x, complex_row_vector y):
    complex_row_vector, 112
  (complex x, complex_vector y):
    complex_vector, 112
  (complex_matrix x, complex y):
    complex_matrix, 113
  (complex_matrix x, complex_matrix y):
    complex_matrix, 113
  (complex_row_vector x, complex y):
    complex_row_vector, 112
  (complex_row_vector x, com-
    plex_row_vector y):  com-
    plex_row_vector, 112
  (complex_vector x, complex y):
    complex_vector, 112
  (complex_vector x, complex_vector y):
    complex_vector, 112
  (matrix x, matrix y):  matrix, 69
  (matrix x, real y):  matrix, 69
  (real x, matrix y):  matrix, 69
  (real x, row_vector y):  row_vector,
    69
  (real x, vector y):  vector, 69
  (row_vector x, real y):  row_vector,
    69
  (row_vector x, row_vector y):
    row_vector, 69
  (vector x, real y):  vector, 69
  (vector x, vector y):  vector, 69
operator_elt_multiply
  (complex_matrix x, complex_matrix y):
    complex_matrix, 112
  (complex_row_vector x, com-
    plex_row_vector y):  com-
    plex_row_vector, 112
  (complex_vector x, complex_vector y):
    complex_vector, 112
  (matrix x, matrix y):  matrix, 69
  (row_vector x, row_vector y):
    row_vector, 68
  (vector x, vector y):  vector, 68
operator_elt_pow
  (complex_matrix x, complex y):
    matrix, 114
  (complex_matrix x, complex_matrix y):
    matrix, 113
  (complex x, complex_matrix y):
    matrix, 114
  (complex x, complex_row_vector y):
    complex_row_vector, 113
  (complex x, complex_vector y):
    vector, 113
  (complex_row_vector x, complex y):
    complex_row_vector, 113
  (complex_row_vector x, com-
    plex_row_vector y):  com-
    plex_row_vector, 113
  (complex_vector x, complex y):
    vector, 113
  (complex_vector x, complex_vector y):
    vector, 113
  (matrix x, matrix y):  matrix, 70
  (matrix x, real y):  matrix, 70
  (real x, matrix y):  matrix, 70
  (real x, row_vector y):  row_vector,
    70
  (real x, vector y):  vector, 70
  (row_vector x, real y):  row_vector,
    70
  (row_vector x, row_vector y):
    row_vector, 70
  (vector x, real y):  vector, 70
  (vector x, vector y):  vector, 70
operator_int_divide
  (int x, int y):  int, 7
operator_left_div
  (matrix A, matrix B): matrix, 95
  (matrix A, vector b): vector, 95
operator_logical_and
  (int x, int y):  int, 20
  (real x, real y):  int, 20
operator_logical_equal
  (complex x, complex y):  int, 46
  (int x, int y):  int, 19
  (real x, real y):  int, 19
operator_logical_greater_than
  (int x, int y):  int, 18
  (real x, real y):  int, 18
operator_logical_greater_than_equal
  (int x, int y):  int, 19
  (real x, real y):  int, 19
operator_logical_less_than

```

```

(int x, int y): int,18
(real x, real y): int,18
operator_logical_less_than_equal
(int x, int y): int,18
(real x, real y): int,18
operator_logical_not_equal
(complex x, complex y): int,46
(int x, int y): int,19
(real x, real y): int,19
operator_logical_or
(int x, int y): int,21
(real x, real y): int,21
operator_mod
(int x, int y): int,8
operator_multiply
(complex x, complex y): complex,45
(complex x, complex_matrix y):
    complex_matrix,108
(complex x, complex_vector y):
    complex_vector,108
(complex x, row_complex_vector y):
    row_complex_vector,108
(complex_matrix x, complex y):
    complex_matrix,109
(complex_matrix x, complex_matrix y):
    complex_matrix,109
(complex_matrix x, complex_vector y):
    complex_vector,109
(complex_vector x, complex y):
    complex_vector,109
(complex_vector x,
    row_complex_vector y): complex_matrix,109
(int x, int y): int,7
(matrix x, matrix y): matrix,66
(matrix x, real y): matrix,66
(matrix x, vector y): vector,66
(real x, matrix y): matrix,65
(real x, real y): real,22
(real x, row_vector y): row_vector,
    65
(real x, vector y): vector,65
(row_complex_vector x, complex y):
    row_complex_vector,109
(row_complex_vector x,
    complex_matrix y):
    row_complex_vector,109
(row_complex_vector x, com-
    plex_vector y): complex,109
(row_vector x, matrix y):
    row_vector,66
(row_vector x, real y): row_vector,
    66
(row_vector x, vector y): real,66
(vector x, real y): vector,65
(vector x, row_vector y): matrix,66
operator_negation
(int x): int,20
(real x): int,20
operator_pow
(complex x, complex y): complex,46
(real x, real y): real,23
operator_subtract
(T x): T,8,23,45,64,107
(complex x, complex y): complex,45
(complex x, complex_matrix y):
    complex_matrix,111
(complex x, complex_vector y):
    complex_vector,110
(complex x, row_complex_vector y):
    row_complex_vector,110
(complex_matrix x): complex_matrix,
    107
(complex_matrix x, complex y):
    complex_matrix,111
(complex_matrix x, complex_matrix y):
    complex_matrix,108
(complex_vector x): complex_vector,
    107
(complex_vector x, complex y):
    complex_vector,110
(complex_vector x, complex_vector y):
    complex_vector,108
(int x): int,8
(int x, int y): int,7
(matrix x): matrix,64
(matrix x, matrix y): matrix,65
(matrix x, real y): matrix,67
(real x): real,23
(real x, matrix y): matrix,67
(real x, real y): real,22
(real x, row_vector y): row_vector,
    67
(real x, vector y): vector,67
(row_complex_vector x):
    row_complex_vector,107
(row_complex_vector x, complex y):
    row_complex_vector,110

```

```

(row_complex_vector x,
     row_complex_vector y):
     row_complex_vector, 108
(row_vector x): row_vector, 64
(row_vector x, real y): row_vector,
     67
(row_vector x, row_vector y):
     row_vector, 65
(vector x): vector, 64
(vector x, real y): vector, 67
(vector x, vector y): vector, 65
operator_subtract
(complex z): complex, 45
operator_transpose
(complex_matrix x): complex_matrix,
     111
(complex_vector x):
     row_complex_vector, 111
(matrix x): matrix, 68
(row_complex_vector x): com-
     plex_vector, 111
(row_vector x): vector, 68
(vector x): row_vector, 68
ordered_constraint
(vectors y): vectors, 161
ordered_jacobian
(vectors y): vectors, 161
ordered_logistic
sampling statement, 202
ordered_logistic_glm
sampling statement, 204
ordered_logistic_glm_lpmpf
(array[] int y | matrix x, vector
     beta, vector c): real, 205
(array[] int y | row_vector x, vec-
     tor beta, vector c): real,
     204
(int y | matrix x, vector beta,
     vector c): real, 204
(int y | row_vector x, vector beta,
     vector c): real, 204
ordered_logistic_glm_lupmf
(array[] int y | matrix x, vector
     beta, vector c): real, 205
(array[] int y | row_vector x, vec-
     tor beta, vector c): real,
     204
(int y | matrix x, vector beta,
     vector c): real, 204
ordered_logistic_lpmf
(int y | row_vector x, vector beta,
     vector c): real, 204
ordered_logistic_lpmpf
(ints k | vector eta, vectors c):
     real, 203
ordered_logistic_lupmf
(ints k | vector eta, vectors c):
     real, 203
ordered_logistic_rng
(real eta, vector c): int, 203
ordered_probit
sampling statement, 205
ordered_probit_lpmpf
(ints k | real eta, vectors c):
     real, 206
(ints k | vector eta, vectors c):
     real, 205
ordered_probit_lupmf
(ints k | real eta, vectors c):
     real, 206
(ints k | vector eta, vectors c):
     real, 206
ordered_probit_rng
(real eta, vector c): int, 206
ordered_unconstraint
(vectors x): vectors, 161
owens_t
(T1 x, T2 y): R, 31
(real h, real a): real, 31
pareto
sampling statement, 256
pareto_cdf
(reals y | reals y_min, reals al-
     pha): real, 256
pareto_lccdf
(reals y | reals y_min, reals al-
     pha): real, 257
pareto_lcddf
(reals y | reals y_min, reals al-
     pha): real, 256
pareto_lpdf
(reals y | reals y_min, reals al-
     pha): real, 256
pareto_lupdf
(reals y | reals y_min, reals al-
     pha): real, 256
pareto_rng
(reals y_min, reals alpha): R, 257
pareto_type_2

```

```

sampling statement, 257
pareto_type_2_cdf
  (reals y | reals mu, reals lambda,
   reals alpha): real, 258
pareto_type_2_lccdf
  (reals y | reals mu, reals lambda,
   reals alpha): real, 258
pareto_type_2_lcdf
  (reals y | reals mu, reals lambda,
   reals alpha): real, 258
pareto_type_2_lpdf
  (reals y | reals mu, reals lambda,
   reals alpha): real, 257
pareto_type_2_lupdf
  (reals y | reals mu, reals lambda,
   reals alpha): real, 257
pareto_type_2_rng
  (reals mu, reals lambda, reals
   alpha): R, 258
phi
  (T x): R, 30
phi_approx
  (T x): R, 30
pi
  (): real, 16
poisson
  sampling statement, 213
poisson_cdf
  (ints n | reals lambda): real, 214
poisson_lccdf
  (ints n | reals lambda): real, 214
poisson_lcdf
  (ints n | reals lambda): real, 214
poisson_log
  sampling statement, 215
poisson_log_glm
  sampling statement, 215
poisson_log_glm_lpmf
  (array[] int y | matrix x, real
   alpha, vector beta): real, 217
  (array[] int y | matrix x, vector
   alpha, vector beta): real, 217
  (array[] int y | row_vector x, real
   alpha, vector beta): real, 216
  (array[] int y | row_vector x, vec-
   tor alpha, vector beta): real,
   216
  (int y | matrix x, real alpha,
   vector beta): real, 215
print
  (T1 x1, ..., TN xN): void, 4
prod
  (array[] int x): real, 54
  (array[] real x): real, 54
  (complex_matrix x): complex, 117

```

```

(complex_row_vector x): complex, 117
(complex_vector x): complex, 117
(matrix x): real, 76
(row_vector x): real, 76
(vector x): real, 76
proj
  (complex z): complex, 48
qr
  (matrix A): tuple(matrix, matrix),
    101
qr_q
  (matrix A): matrix, 101
qr_r
  (matrix A): matrix, 101
qr_thin
  (matrix A): tuple(matrix, matrix),
    100
qr_thin_q
  (matrix A): matrix, 100
qr_thin_r
  (matrix A): matrix, 100
quad_form
  (matrix A, matrix B): matrix, 73
  (matrix A, vector B): real, 73
quad_form_diag
  (matrix m, row_vector rv): matrix,
    73
  (matrix m, vector v): matrix, 73
quad_form_sym
  (matrix A, matrix B): matrix, 73
  (matrix A, vector B): real, 73
quantile
  (data array[] real x, data array[]
    real p): real, 57
  (data array[] real x, data real p):
    real, 57
  (data row_vector x, data array[]
    real p): real, 78
  (data row_vector x, data real p):
    real, 78
  (data vector x, data array[] real
    p): real, 77
  (data vector x, data real p): real,
    77
rank
  (array[] int v, int s): int, 61
  (array[] real v, int s): int, 61
  (row_vector v, int s): int, 103
  (vector v, int s): int, 103
rayleigh
  sampling statement, 253
rayleigh_cdf
  (real y | real sigma): real, 254
rayleigh_lccdf
  (real y | real sigma): real, 254
rayleigh_lcdf
  (real y | real sigma): real, 254
rayleigh_lpdf
  (reals y | reals sigma): real, 253
rayleigh_lupdf
  (reals y | reals sigma): real, 253
rayleigh_rng
  (reals sigma): R, 254
reduce_sum
  (F f, array[] T x, int grainsize, T1
    s1, T2 s2, ...): real, 155
reduce_sum_static
  (F f, array[] T x, int grainsize, T1
    s1, T2 s2, ...): real, 155
reject
  (T1 x1, ..., TN xN): void, 4
rep_array
  (T x, int k, int m, int n): array[,,]
    T, 59
  (T x, int m, int n): array[,] T, 58
  (T x, int n): array[] T, 58
rep_matrix
  (complex z, int m, int n): complex_matrix, 118
  (complex_row_vector rv, int m):
    complex_matrix, 118
  (complex_vector v, int n): complex_matrix, 118
  (real x, int m, int n): matrix, 78
  (row_vector rv, int m): matrix, 78
  (vector v, int n): matrix, 78
rep_row_vector
  (complex z, int n): complex_row_vector, 118
  (real x, int n): row_vector, 78
rep_vector
  (complex z, int m): complex_vector,
    118
  (real x, int m): vector, 78
reverse
  (array[] T v): array[] T, 62
  (complex_row_vector v): complex_row_vector, 128

```

```

(complex_vector v): complex_vector,
  128
(row_vector v): row_vector, 104
(vector v): vector, 104
rising_factorial
  (T1 x, T2 y): R, 37
  (real x, int n): real, 37
round
  (T x): R, 26
row
  (complex_matrix x, int m): complex_row_vector, 119
  (matrix x, int m): row_vector, 82
rows
  (complex_matrix x): int, 106
  (complex_row_vector x): int, 106
  (complex_vector x): int, 106
  (matrix x): int, 63
  (row_vector x): int, 63
  (vector x): int, 63
rows_dot_product
  (complex_matrix x, complex_matrix y): complex_vector, 115
  (complex_row_vector x, complex_row_vector y): complex_vector, 115
  (complex_vector x, complex_vector y): complex_vector, 115
  (matrix x, matrix y): vector, 71
  (row_vector x, row_vector y): vector, 71
  (vector x, vector y): vector, 71
rows_dot_self
  (complex_matrix x): complex_vector, 116
  (complex_row_vector x): complex_vector, 115
  (complex_vector x): complex_vector, 115
  (matrix x): vector, 72
  (row_vector x): vector, 72
  (vector x): vector, 72
scale_matrix_exp_multiply
  (real t, matrix A, matrix B): matrix, 97
scaled_inv_chi_square
  sampling statement, 246
scaled_inv_chi_square_cdf
  (reals y | reals nu, reals sigma):
    real, 247
scaled_inv_chi_square_lccdf
  (reals y | reals nu, reals sigma):
    real, 247
scaled_inv_chi_square_lcdf
  (reals y | reals nu, reals sigma):
    real, 247
scaled_inv_chi_square_lpdf
  (reals y | reals nu, reals sigma):
    real, 246
scaled_inv_chi_square_lupdf
  (reals y | reals nu, reals sigma):
    real, 247
scaled_inv_chi_square_rng
  (reals nu, reals sigma): R, 247
sd
  (array[] real x): real, 55
  (matrix x): real, 77
  (row_vector x): real, 77
  (vector x): real, 77
segment
  (array[] T sv, int i, int n): array[] T, 84
  (complex_row_vector rv, int i, int n): complex_row_vector, 121
  (complex_vector v, int i, int n): complex_vector, 120
  (row_vector rv, int i, int n): row_vector, 84
  (vector v, int i, int n): vector, 84
simplex_constrain
  (vectors y): vectors, 162
simplex_jacobian
  (vectors y): vectors, 162
simplex_unconstrain
  (vectors x): vectors, 162
sin
  (T x): R, 28
  (complex z): complex, 50
singular_values
  (complex_matrix A): vector, 126
  (matrix A): vector, 102
sinh
  (T x): R, 29
  (complex z): complex, 51
size
  (array[] T x): int, 58
  (complex_matrix x): int, 107
  (complex_row_vector x): int, 107

```

```

(complex_vector x): int,107
(int x): int,9
(matrix x): int,64
(real x): int,9
(row_vector x): int,64
(vector x): int,64
skew_double_exponential
    sampling statement,241
skew_double_exponential_cdf
    (reals y | reals mu, reals sigma,
     reals tau): real,241
skew_double_exponential_lccdf
    (reals y | reals mu, reals sigma,
     reals tau): real,242
skew_double_exponential_lcdf
    (reals y | reals mu, reals sigma,
     reals tau): real,241
skew_double_exponential_lpdf
    (reals y | reals mu, reals sigma,
     reals tau): real,241
skew_double_exponential_lupdf
    (reals y | reals mu, reals sigma,
     reals tau): real,241
skew_double_exponential_rng
    (reals mu, reals sigma, reals tau):
        R,242
skew_normal
    sampling statement,233
skew_normal_cdf
    (reals y | reals xi, reals omega,
     reals alpha): real,233
skew_normal_lccdf
    (reals y | reals xi, reals omega,
     reals alpha): real,234
skew_normal_lcdf
    (reals y | reals xi, reals omega,
     reals alpha): real,234
skew_normal_lpdf
    (reals y | reals xi, reals omega,
     reals alpha): real,233
skew_normal_lupdf
    (reals y | reals xi, reals omega,
     reals alpha): real,233
skew_normal_rng
    (reals xi, reals omega, real alpha):
        R,234
softmax
    (vector x): vector,86
solve_newton
    (function algebra_system, vector
     y_guess, ...): vector,143
solve_newton_tol
    (function algebra_system, vec-
     tor y_guess, data real scal-
     ing_step, data real f_tol, int
     max_steps, ...): vector,143
solve_powell
    (function algebra_system, vector
     y_guess, ...): vector,143
solve_powell_tol
    (function algebra_system, vector
     y_guess, data real rel_tol,
     data real f_tol, int max_steps,
     ...): vector,143
sort_asc
    (array[] int v): array[] int,61
    (array[] real v): array[] real,61
    (row_vector v): row_vector,103
    (vector v): vector,102
sort_desc
    (array[] int v): array[] int,61
    (array[] real v): array[] real,61
    (row_vector v): row_vector,103
    (vector v): vector,103
sort_indices_asc
    (array[] int v): array[] int,61
    (array[] real v): array[] int,61
    (row_vector v): array[] int,103
    (vector v): array[] int,103
sort_indices_desc
    (array[] int v): array[] int,61
    (array[] real v): array[] int,61
    (row_vector v): array[] int,103
    (vector v): array[] int,103
sqrt
    (T x): R,26
    (complex x): complex,50
sqrt2
    (): real,16
square
    (T x): R,26
squared_distance
    (row_vector x, row_vector y): real,
        57
    (row_vector x, vector y): real,57
    (vector x, row_vector y): real,57
    (vector x, vector y): real,56
std_normal

```

```

sampling statement, 227
std_normal_cdf
  (reals y): real, 227
std_normal_lccdf
  (reals y): real, 227
std_normal_lcdf
  (reals y): real, 227
std_normal_log_qf
  (T x): R, 227
std_normal_lpdf
  (reals y): real, 227
std_normal_lupdf
  (reals y): real, 227
std_normal_qf
  (T x): R, 227
std_normal_rng
  (): real, 228
step
  (real x): real, 21
stochastic_column_constrain
  (matrices y): matrices, 166
stochastic_column_jacobian
  (matrices y): matrices, 166
stochastic_column_unconstrain
  (matrices x): matrices, 166
stochastic_row_constrain
  (matrices y): matrices, 167
stochastic_row_jacobian
  (matrices y): matrices, 167
stochastic_row_unconstrain
  (matrices x): matrices, 167
student_t
  sampling statement, 234
student_t_cdf
  (reals y | reals nu, reals mu, reals
  sigma): real, 235
student_t_lccdf
  (reals y | reals nu, reals mu, reals
  sigma): real, 235
student_t_lcdf
  (reals y | reals nu, reals mu, reals
  sigma): real, 235
student_t_lpdf
  (reals y | reals nu, reals mu, reals
  sigma): real, 235
student_t_lupdf
  (reals y | reals nu, reals mu, reals
  sigma): real, 235
student_t_rng
  (reals nu, reals mu, reals sigma):
  R, 235
sub_col
  (complex_matrix x, int i, int j, int
  n_rows): complex_vector, 120
  (matrix x, int i, int j, int
  n_rows): vector, 83
sub_row
  (complex_matrix x, int i, int j, int
  n_cols): complex_row_vector,
  120
  (matrix x, int i, int j, int
  n_cols): row_vector, 83
sum
  (array[] complex x): complex, 53
  (array[] int x): int, 53
  (array[] real x): real, 53
  (complex_matrix x): complex, 117
  (complex_row_vector x): complex, 116
  (complex_vector x): complex, 116
  (matrix x): real, 76
  (row_vector x): real, 76
  (vector x): real, 76
sum_to_zero_constrain
  (matrices y): matrices, 167
  (vectors y): vectors, 162
sum_to_zero_jacobian
  (matrices y): matrices, 167
  (vectors y): vectors, 163
sum_to_zero_unconstrain
  (matrices x): matrices, 168
  (vectors x): vectors, 163
svd
  (complex_matrix A): tuple(
    complex_matrix, vector,
    complex_matrix), 126
  (matrix A): tuple(matrix, vector,
  matrix), 102
svd_U
  (complex_matrix A): complex_matrix,
  126
  (matrix A): matrix, 102
svd_V
  (complex_matrix A): complex_matrix,
  126
  (matrix A): matrix, 102
symmetrize_from_lower_tri
  (complex_matrix A): complex_matrix,
  119

```

```

(matrix A): matrix, 79
tail
  (array[] T sv, int n): array[] T, 83
  (complex_row_vector rv, int n):
    complex_row_vector, 120
  (complex_vector v, int n): complex_vector, 120
  (row_vector rv, int n): row_vector, 83
  (vector v, int n): vector, 83
tan
  (T x): R, 28
  (complex z): complex, 50
tanh
  (T x): R, 29
  (complex z): complex, 51
target
  (): real, 17
tcrossprod
  (matrix x): matrix, 72
tgamma
  (T x): R, 32
to_array_1d
  (array[...] complex a): array[] complex, 138
  (array[...] int a): array[] int, 138
  (array[...] real a): array[] real, 138
  (complex_matrix m): array[] complex, 138
  (complex_row_vector v): array[] complex, 138
  (complex_vector v): array[] complex, 138
  (matrix m): array[] real, 138
  (row_vector v): array[] real, 138
  (vector v): array[] real, 138
to_array_2d
  (complex_matrix m): array[,] complex, 137
  (matrix m): array[,] real, 137
to_complex
  (): complex, 44
  (T1 re, T2 im): z, 44
  (real re): complex, 44
  (real re, real im): complex, 44
to_int
  (data R x): I, 10
  (data real x): int, 10
to_matrix
  (array[,] complex a): complex_matrix, 135
  (array[,] int a): matrix, 135
  (array[,] real a): matrix, 135
  (array[] complex a, int m, int n):
    complex_matrix, 134
  (array[] complex a, int m, int n,
    int col_major): complex_matrix, 135
  (array[] complex_row_vector vs):
    complex_matrix, 135
  (array[] int a, int m, int n):
    matrix, 134
  (array[] int a, int m, int n, int
    col_major): matrix, 134
  (array[] real a, int m, int n):
    matrix, 134
  (array[] real a, int m, int n, int
    col_major): matrix, 134
  (array[] row_vector vs): matrix, 135
  (complex_matrix A, int m, int n, int
    col_major): complex_matrix, 133
  (complex_matrix M, int m, int n):
    complex_matrix, 132
  (complex_matrix m): complex_matrix,
    132
  (complex_row_vector v): complex_matrix, 132
  (complex_row_vector v, int m, int
    n): complex_matrix, 133
  (complex_row_vector v, int m, int n,
    int col_major): complex_matrix, 134
  (complex_vector v): complex_matrix,
    132
  (complex_vector v, int m, int n):
    complex_matrix, 133
  (complex_vector v, int m, int n, int
    col_major): complex_matrix, 133
  (matrix A, int m, int n, int
    col_major): matrix, 133
  (matrix M, int m, int n): matrix,
    132
  (matrix m): matrix, 132
  (row_vector v): matrix, 132
  (row_vector v, int m, int n): matrix, 133

```

```

(row_vector v, int m, int n, int
    col_major): matrix, 134
(vector v): matrix, 132
(vector v, int m, int n): matrix,
    133
(vector v, int m, int n, int
    col_major): matrix, 133
to_row_vector
    (array[] complex a): com-
        plex_row_vector, 137
    (array[] int a): row_vector, 137
    (array[] real a): row_vector, 137
    (complex_matrix m): com-
        plex_row_vector, 136
    (complex_row_vector v): com-
        plex_row_vector, 137
    (complex_vector v): com-
        plex_row_vector, 137
    (matrix m): row_vector, 136
    (row_vector v): row_vector, 137
    (vector v): row_vector, 137
to_vector
    (array[] complex a): com-
        plex_vector, 136
    (array[] int a): vector, 136
    (array[] real a): vector, 136
    (complex_matrix m): complex_vector,
        136
    (complex_row_vector v): com-
        plex_vector, 136
    (complex_vector v): complex_vector,
        136
    (matrix m): vector, 135
    (row_vector v): vector, 136
    (vector v): vector, 136
trace
    (complex_matrix A): complex, 125
    (matrix A): real, 97
trace_gen_quad_form
    (matrix D, matrix A, matrix B):
        real, 74
trace_quad_form
    (matrix A, matrix B): real, 73
    (matrix A, vector B): real, 73
trigamma
    (T x): R, 33
trunc
    (T x): R, 26
uniform
    sampling statement, 271
uniform_cdf
    (reals y | reals alpha, reals beta):
        real, 271
uniform_lccdf
    (reals y | reals alpha, reals beta):
        real, 272
uniform_lcdf
    (reals y | reals alpha, reals beta):
        real, 271
uniform_lpdf
    (reals y | reals alpha, reals beta):
        real, 271
uniform_lupdf
    (reals y | reals alpha, reals beta):
        real, 271
uniform_rng
    (reals alpha, reals beta): R, 272
uniform_simplex
    (int n): vector, 82
unit_vectors_constrain
    (vectors y): vectors, 163
unit_vectors_jacobian
    (vectors y): vectors, 163
unit_vectors_unconstrain
    (vectors x): vectors, 163
upper_bound constrain
    (reals y, reals ub): reals, 159
upper_bound_jacobian
    (reals x, reals ub): reals, 159
upper_bound_unconstrain
    (reals x, reals ub): reals, 159
variance
    (array[] real x): real, 54
    (matrix x): real, 77
    (row_vector x): real, 77
    (vector x): real, 76
von_mises
    sampling statement, 268
von_mises_cdf
    (reals y | reals mu, reals kappa):
        real, 269
von_mises_lcdf
    (reals y | reals mu, reals kappa):
        real, 269
von_mises_lpdf
    (reals y | reals mu, reals kappa):
        real, 268
von_mises_lupdf

```

```

(real y | reals mu, reals kappa):
    real,269
von_mises_rng
    (reals mu, reals kappa): R,269
weibull
    sampling statement,251
weibull_cdf
    (reals y | reals alpha, reals
        sigma): real,251
weibull_lccdf
    (reals y | reals alpha, reals
        sigma): real,252
weibull_lcdf
    (reals y | reals alpha, reals
        sigma): real,251
weibull_lpdf
    (reals y | reals alpha, reals
        sigma): real,251
weibull_lupdf
    (reals y | reals alpha, reals
        sigma): real,251
weibull_rng
    (reals alpha, reals sigma): R,252
wiener
    sampling statement,261
wiener_lccdf_unnorm
    (real y, real alpha, real tau, real
        beta, real delta): real,263
    (real y, real alpha, real tau,
        real beta, real delta, real
        var_delta, real var_beta, real
        var_tau): real,263
wiener_lcdf_unnorm
    (real y, real alpha, real tau, real
        beta, real delta): real,263
    (real y, real alpha, real tau,
        real beta, real delta, real
        var_delta, real var_beta, real
        var_tau): real,263
wiener_lpdf
    (real y | real alpha, real tau,
        real beta, real delta, real
        var_delta): real,261
    (real y | real alpha, real tau,
        real beta, real delta, real
        var_delta, real var_beta, real
        var_tau): real,261
    (reals y | reals alpha, reals tau,
        reals beta, reals delta): real,

```

[261](#)

```

wiener_lupdf
    (real y | real alpha, real tau,
        real beta, real delta, real
        var_delta): real,262
    (real y | real alpha, real tau,
        real beta, real delta, real
        var_delta, real var_beta, real
        var_tau): real,262
    (reals y | reals alpha, reals tau,
        reals beta, reals delta): real,

```

[262](#)

```

wishart
    sampling statement,293
wishart_cholesky_lpdf
    (matrix L_W | real nu, matrix L_S):
        real,294
wishart_lpdf
    (matrix W | real nu, matrix Sigma):
        real,293
wishart_lupdf
    (matrix L_W | real nu, matrix L_S):
        real,295
    (matrix W | real nu, matrix Sigma):
        real,293
wishart_rng
    (real nu, matrix L_S): matrix,295
    (real nu, matrix Sigma): matrix,294
zeros_array
    (int n): array[] real,81
zeros_int_array
    (int n): array[] int,81
zeros_row_vector
    (int n): row_vector,82
zeros_vector
    (int n): vector,82

```